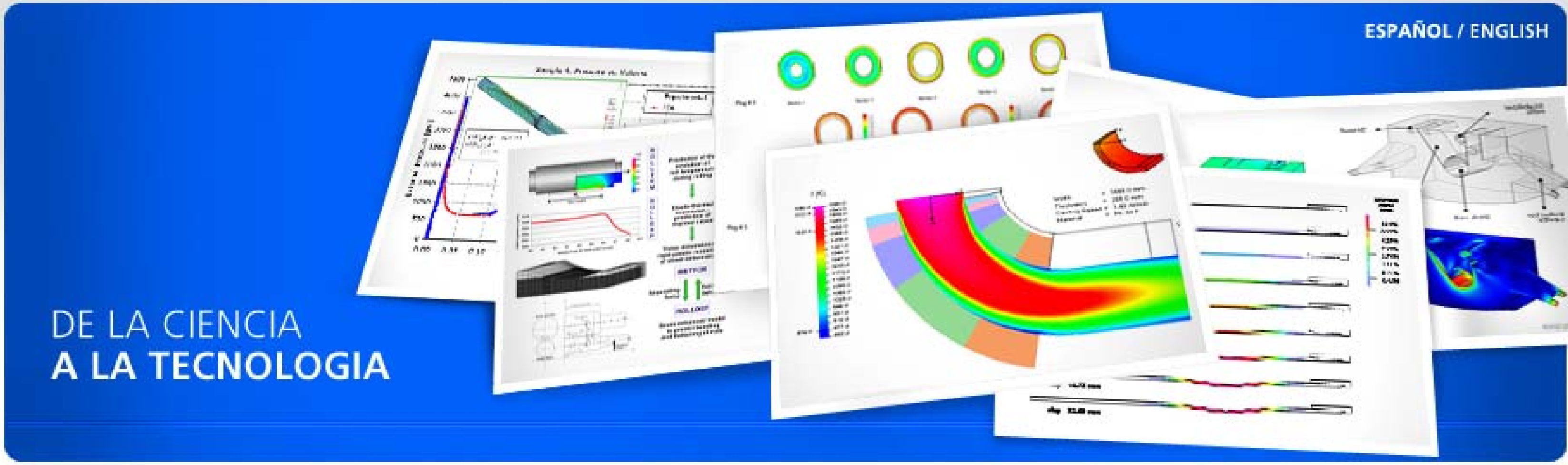
SIM&TEC

Simulación y Tecnología

Simulation and Technology



Advanced Topics in Computational Solid Mechanics Industrial Applications

Section 1: Introduction

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Stanford University
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Winter Quarter 2010

The methodology

Physical Problem

Mathematical Model
(PDE + BC + IC)

Numerical Model

Results verification
and validation

Linear and nonlinear problems

In the analysis of a solid under mechanical and thermal loads
some of the nonlinearities that we may encounter
when formulating the mathematical model are:

Geometrical nonlinearities

The equilibrium equations have to be satisfied in the unknown deformed configuration of the solid rather than in the known unloaded configuration.

Geometrical nonlinearities

When the analyst expects that for her/his purposes the difference between the deformed and unloaded configurations can be neglected she/he may disregard this source of nonlinearity obtaining an important simplification in the mathematical model.

Geometrical nonlinearities

An intermediate step would be to consider the equilibrium in the deformed configuration but to assume that the strains are very small (infinitesimal strains assumption). This also produces an important simplification in the mathematical model.

Geometrical nonlinearities

Of course, all the simplifications introduced in the mathematical model have to be checked for their properness when examining the numerical results.

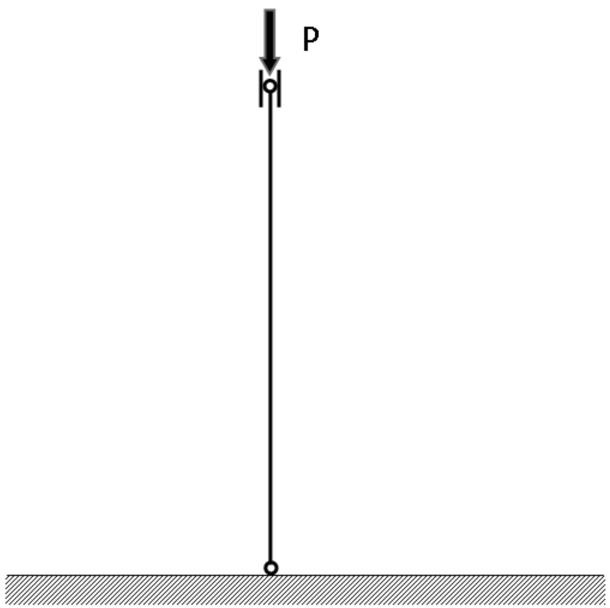
Material nonlinearities

The material stress-strain relation is non linear.

E.g. plasticity, viscoplasticity, creep, fracturing materials (concrete), etc.

Geometrical nonlinearities (Ex. 1)

Infinitesimal strains: buckling of a straight column



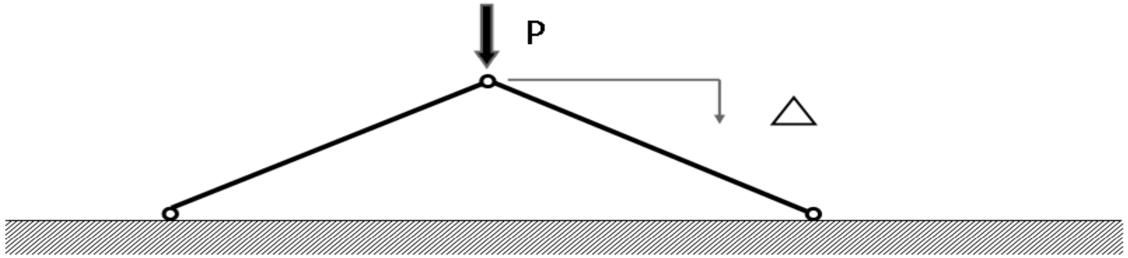
Geometrical nonlinearities (Ex. 1)

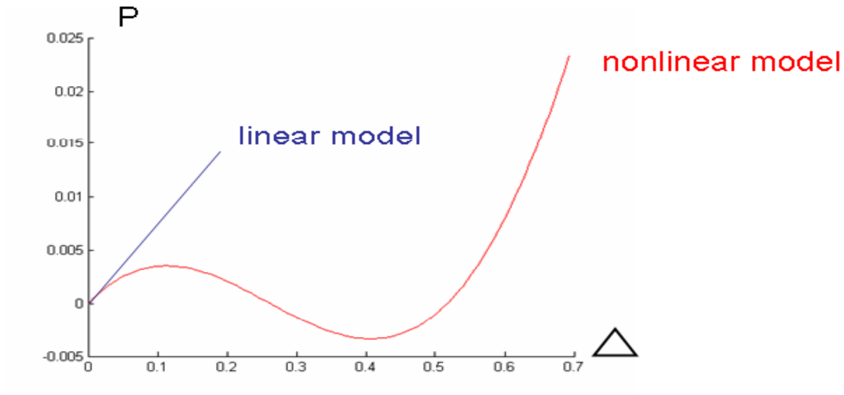
Infinitesimal strains : buckling of a straight column

Even if we decide to model the material as "infinitely" linear elastic the geometrically linear model can only be used for very small displacement and a geometrically nonlinear analysis under the infinitesimal strains assumption should be otherwise performed if we need to predict the column buckling load

Geometrical nonlinearities (Ex. 2)

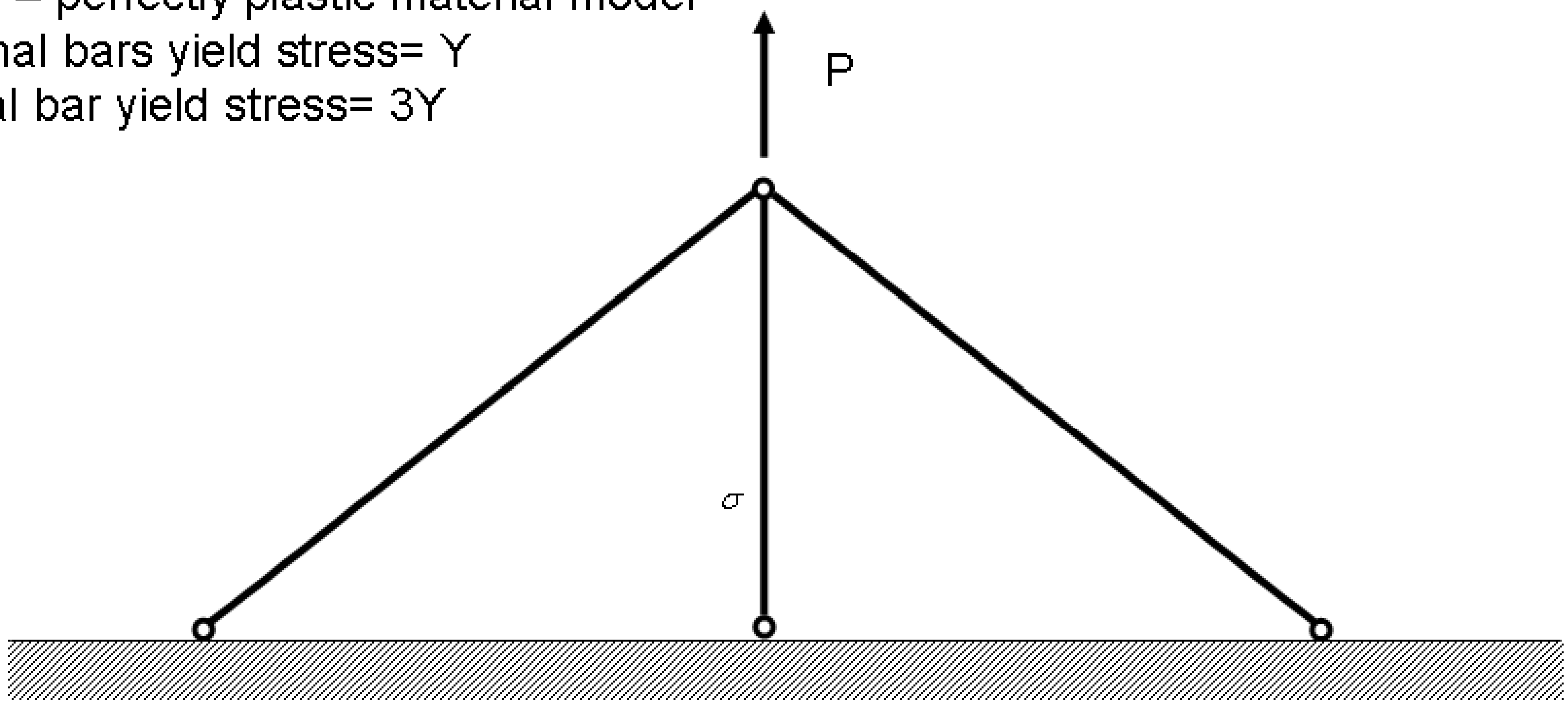
Infinitesimal strains





Geometrical nonlinearities (Ex. 3)

Elastic – perfectly plastic material model
Diagonal bars yield stress= Y
Vertical bar yield stress= 3Y



Geometrical nonlinearities (Ex. 3)

Infinitesimal strains

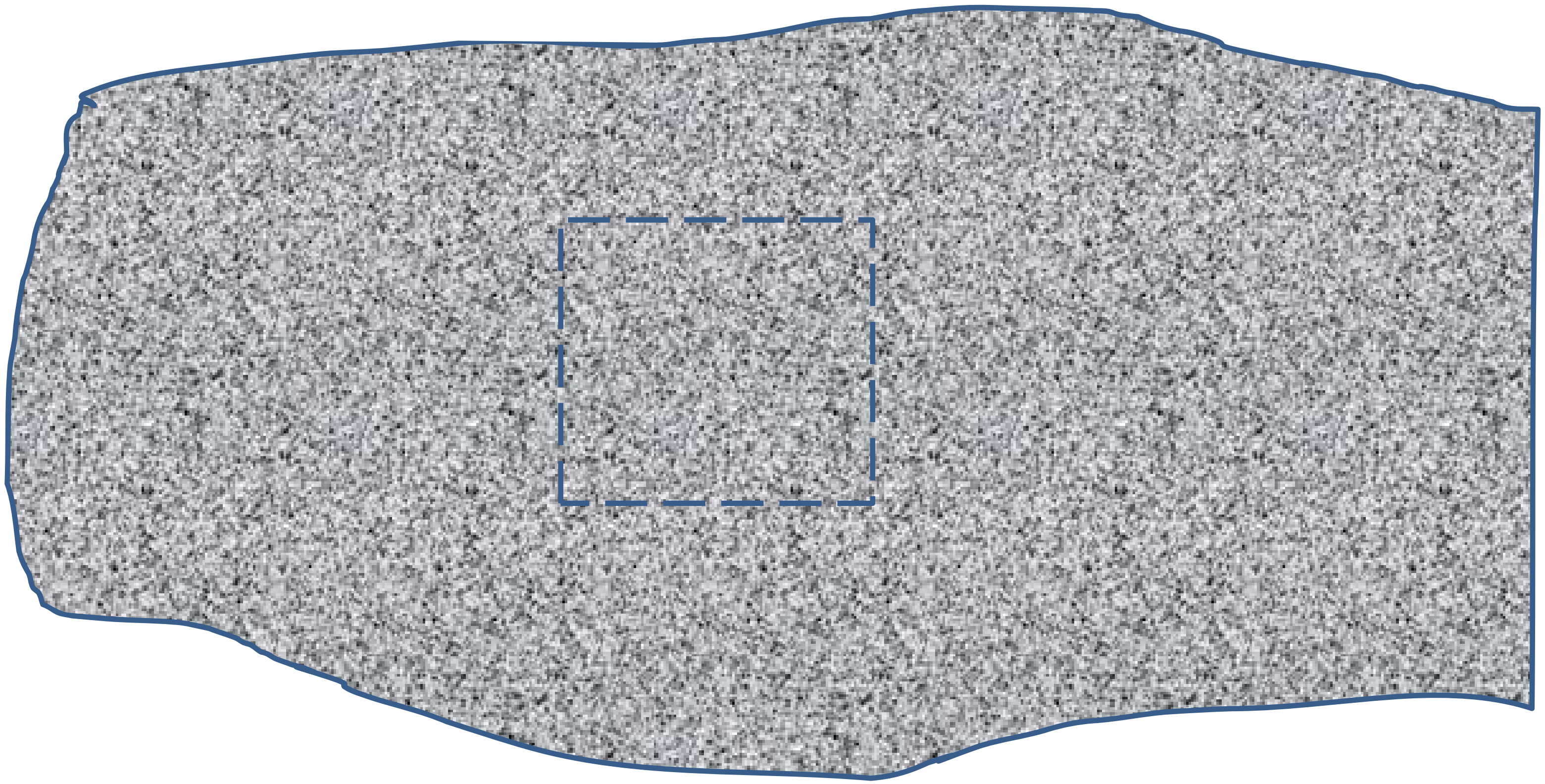
The expected displacements are very small and no equilibrium bifurcation can be expected.

However, after the yielding of the side bars, we can only get a solution if the geometrical nonlinearities, under the infinitesimal strains assumption, are included in the model.

Geometrical nonlinearities (Ex. 4)

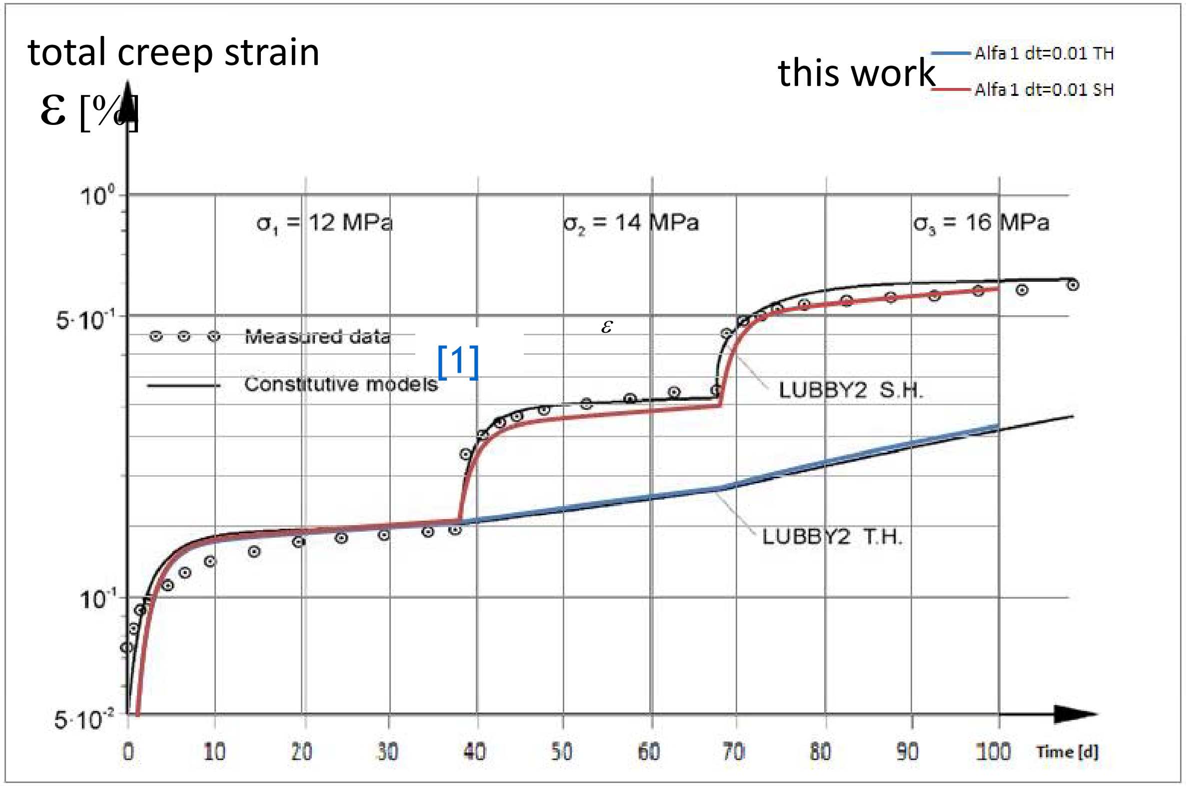
Infinitesimal strains

Material nonlinear only (MNO) (Ex. 5)



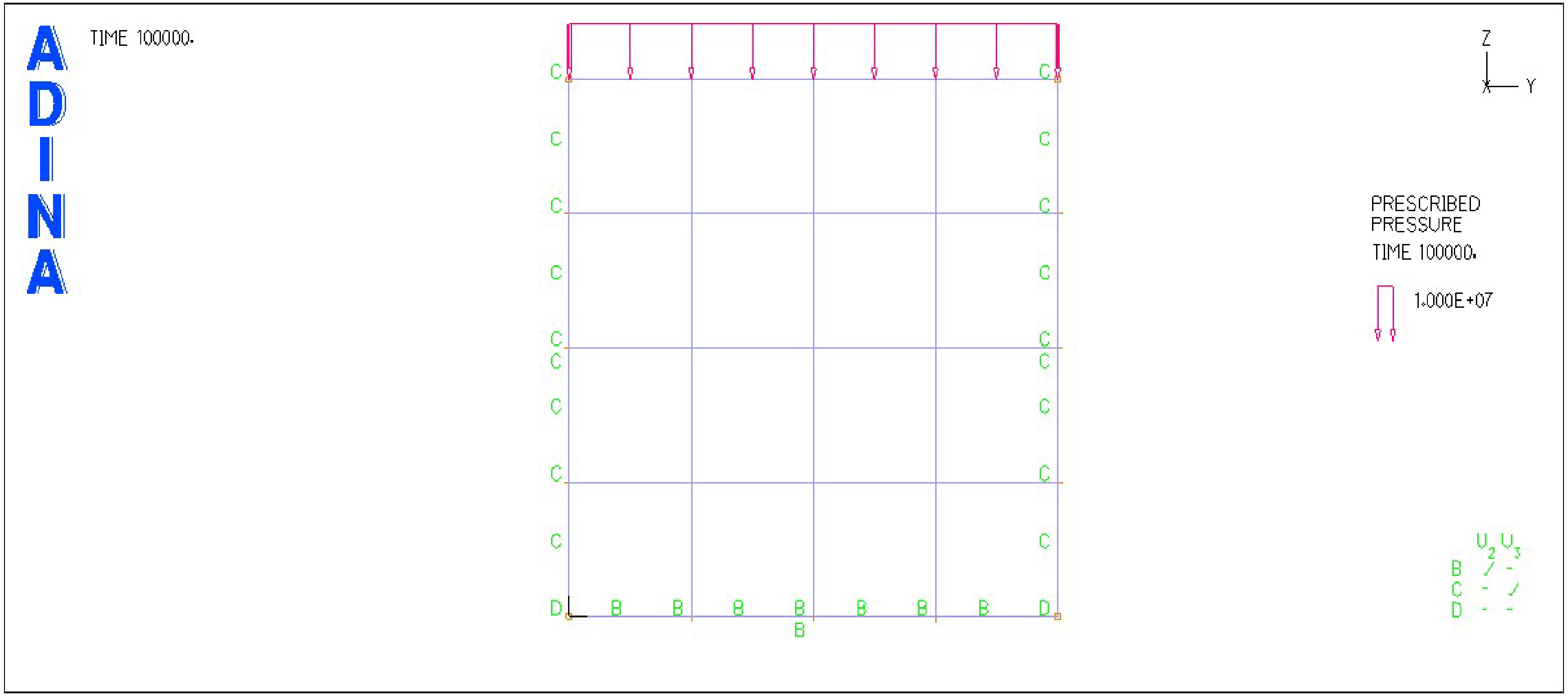
Salt dome

Material nonlinear only (MNO) (Ex. 5)

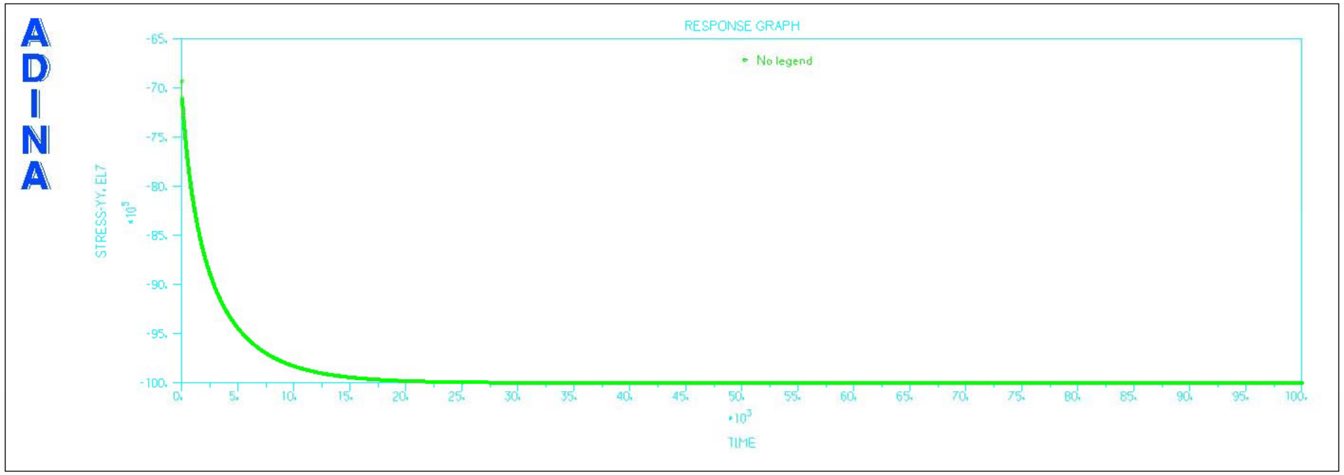


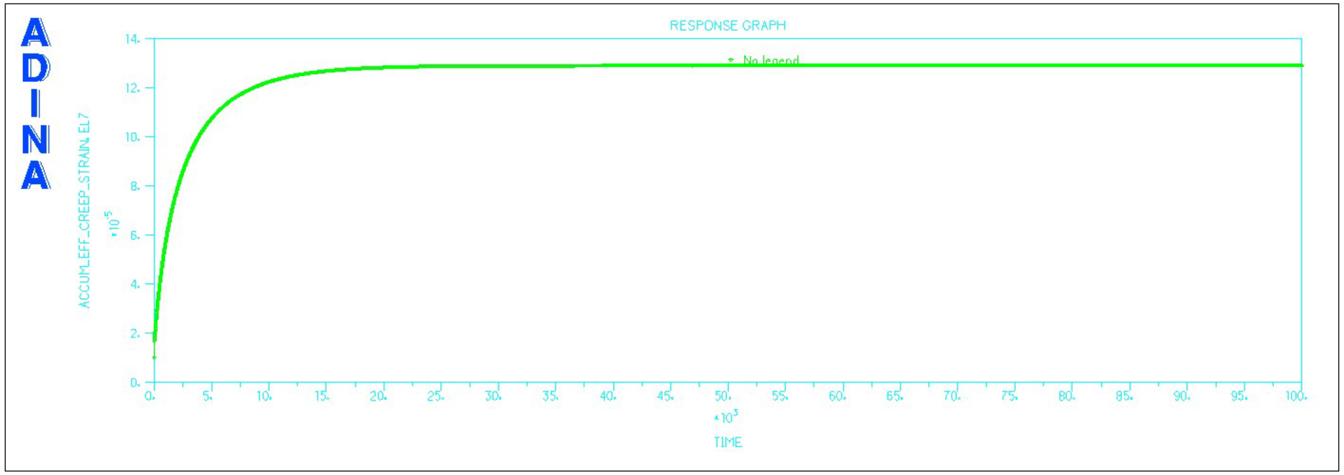
Calibration of creep constitutive models with 1D lab tests

Material nonlinear only (MNO) (Ex. 5)

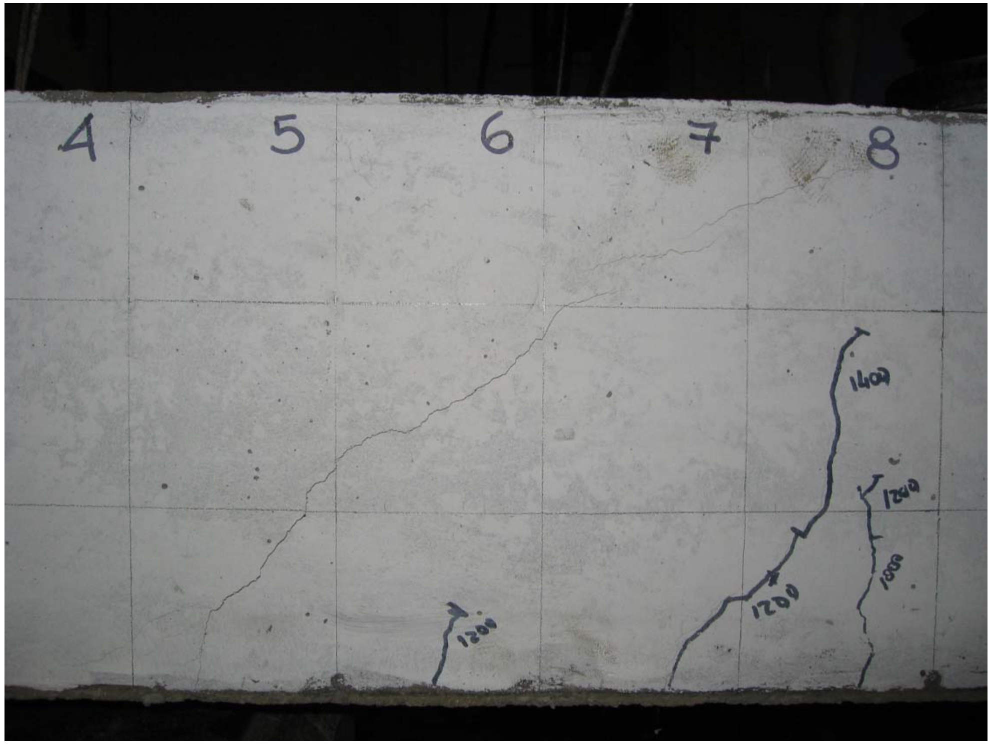


Material nonlinear only (MNO) (Ex. 5)





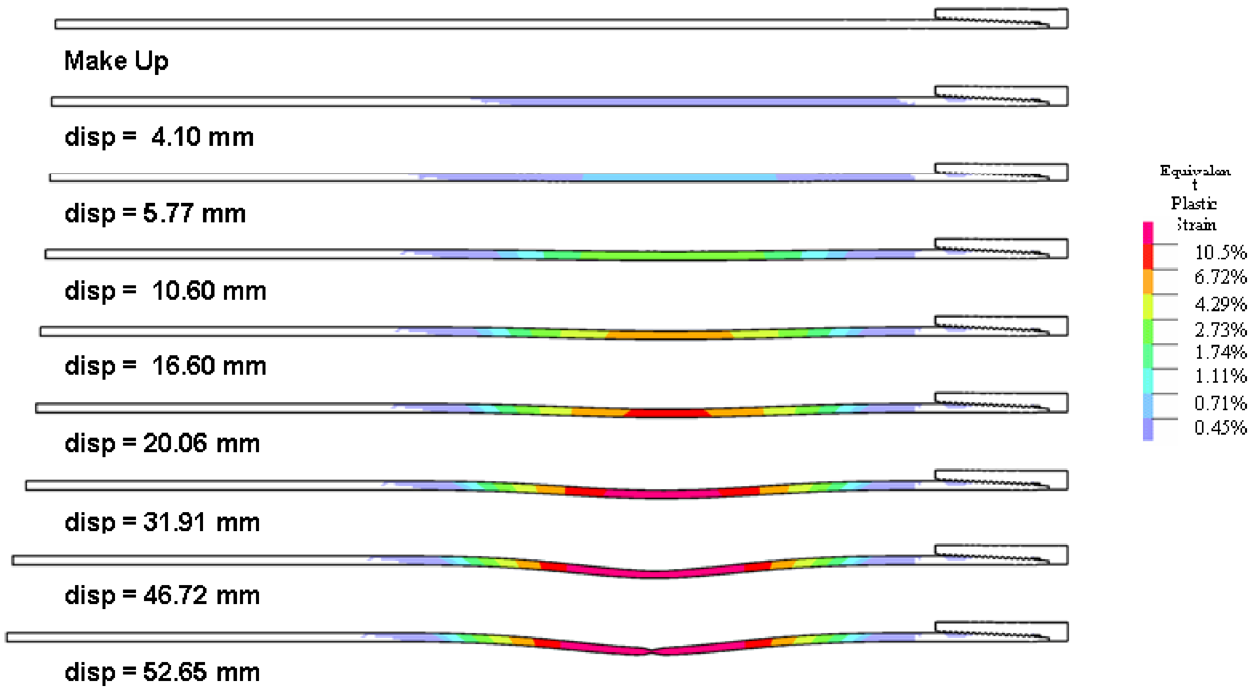
Material nonlinear only (Ex. 6)



Fracturing material: concrete

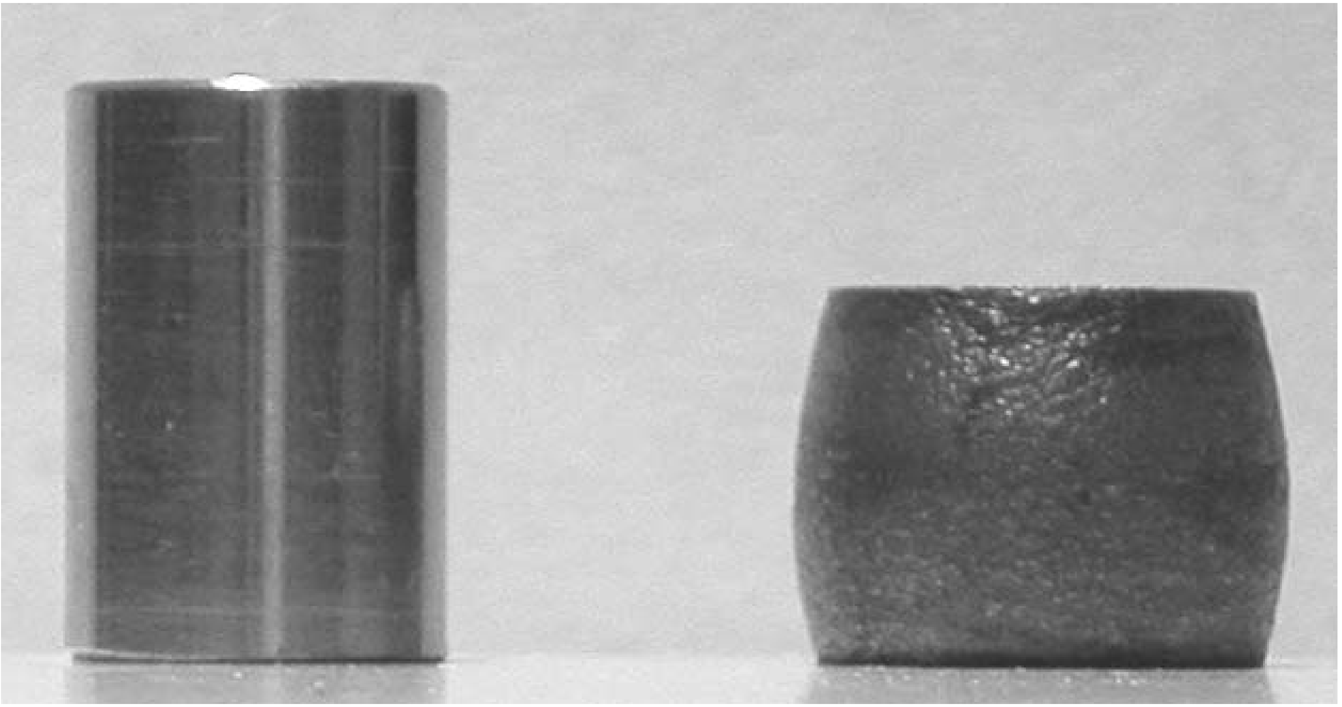
Geometrical and material nonlinearities(Ex. 6)

Finite strains



Geometrical and material nonlinearities(Ex. 7)

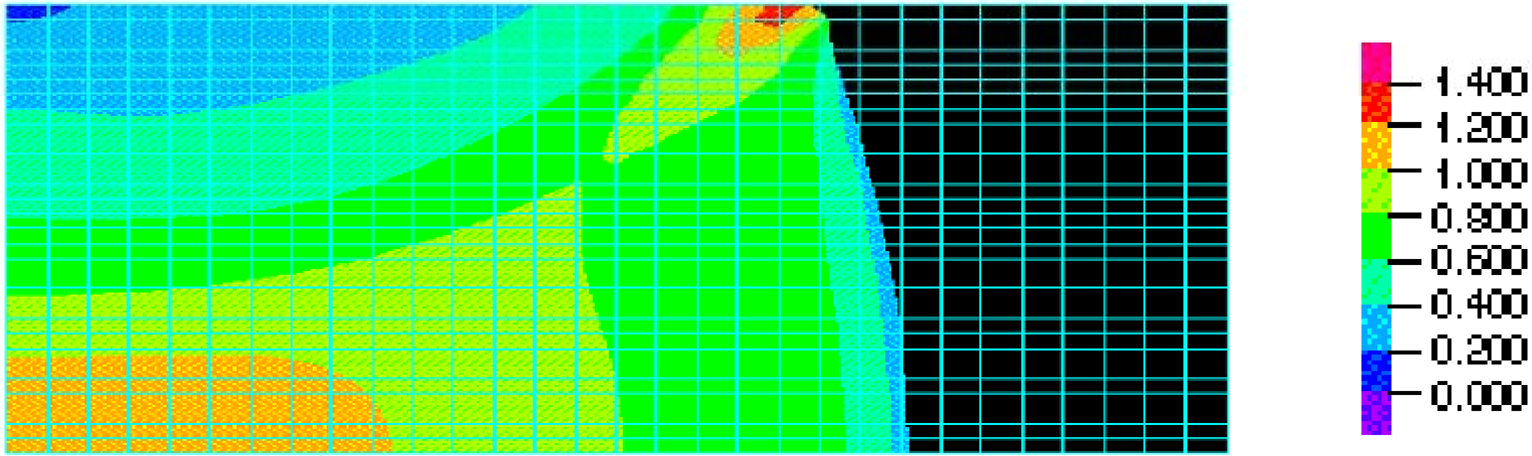
Finite strains



Undeformed sample | Deformed sample

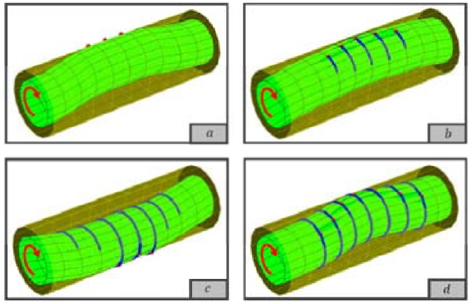
Geometrical and material nonlinearities(Ex. 7)

Finite strains

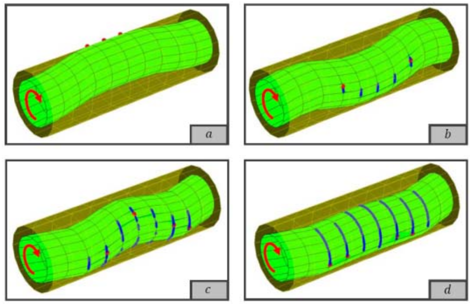


Equivalent plastic strains

Geometrical nonlinear analysis: contact (Ex. 8)



μ (pipes/well)=0.0



μ (pipes/well)=0.1

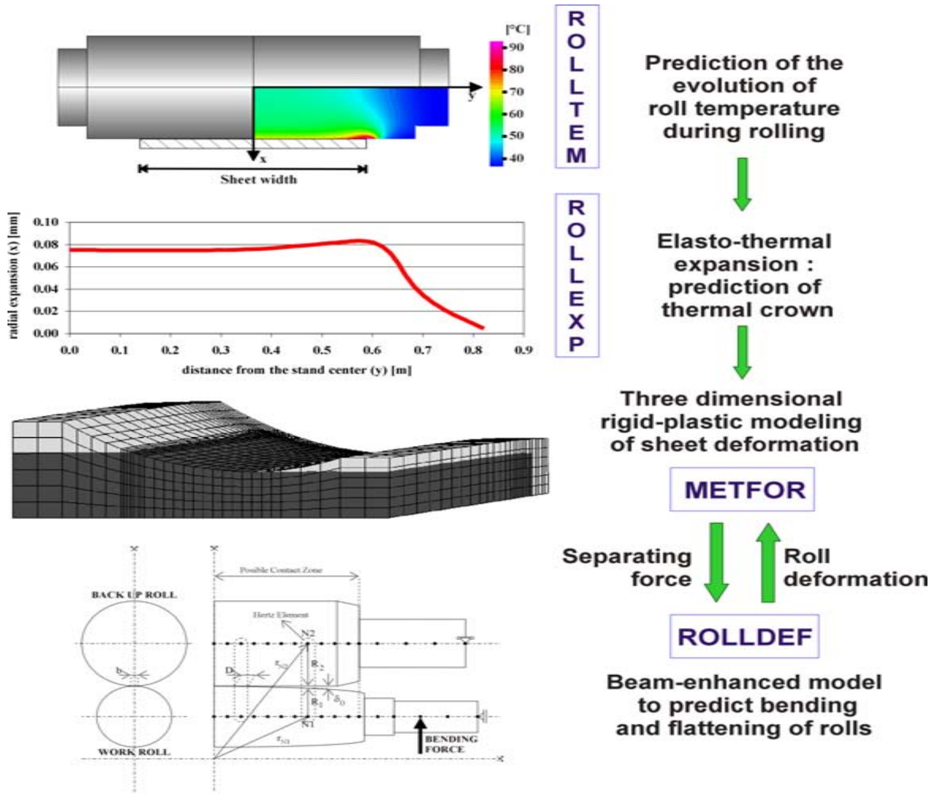
Comparison at the central cross section

Industrial applications
3D Modeling of steel hot rolling

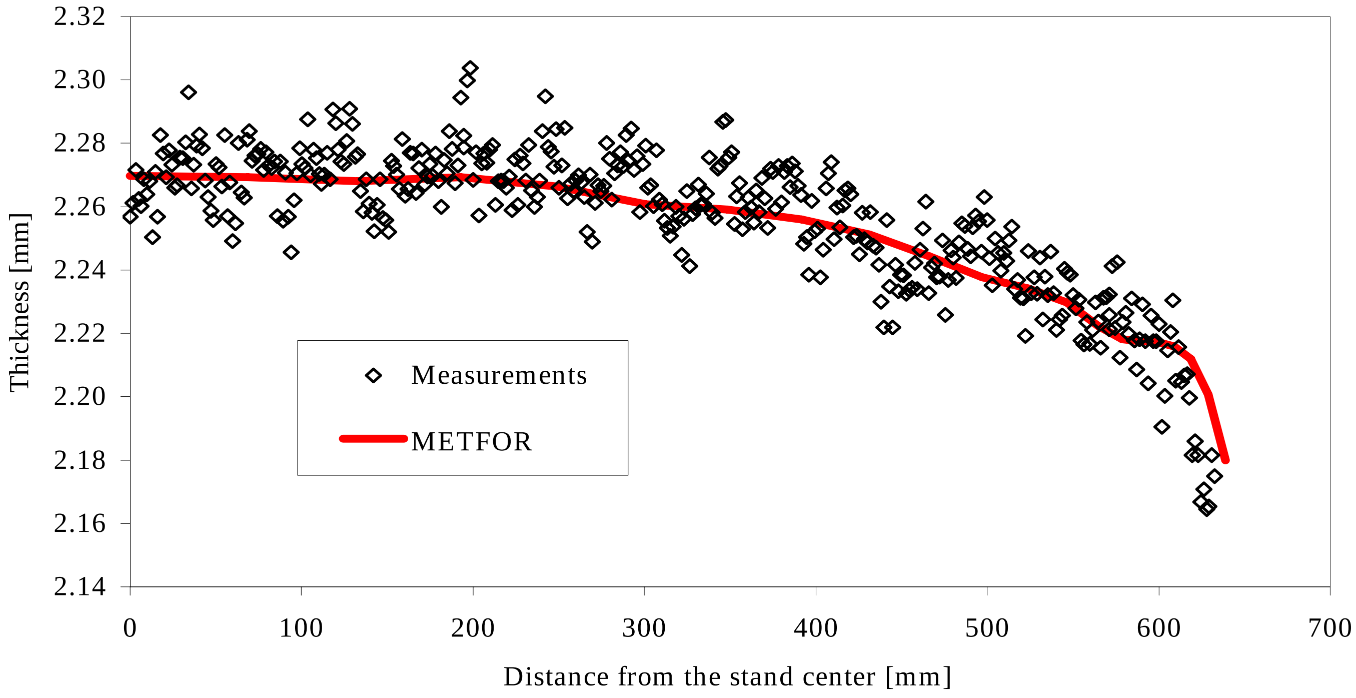
Couple:

- Eulerian formulation that describes the rolled steel deformation

- Standard Lagrangian formulation that describes the rolls deformation

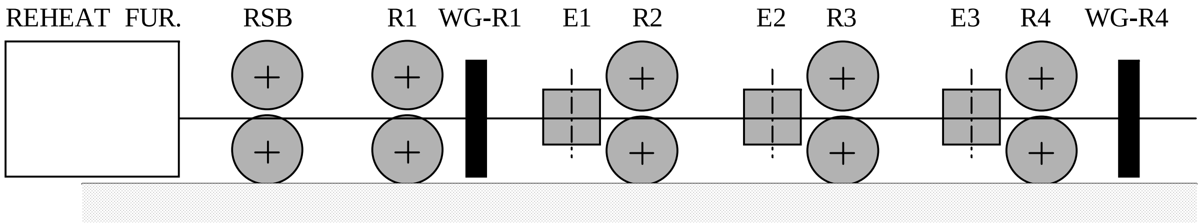


Industrial applications
3D Modeling of steel hot rolling

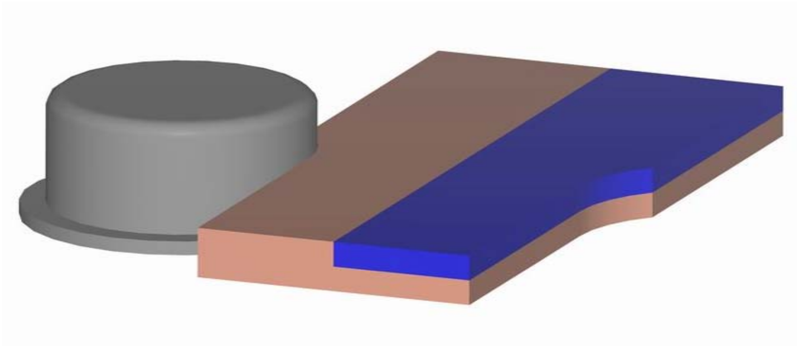


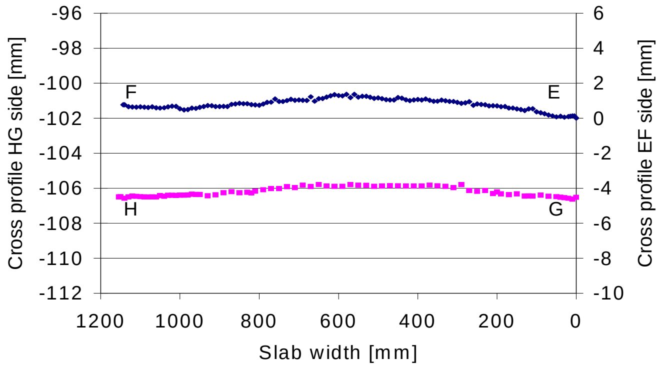
Model Validation

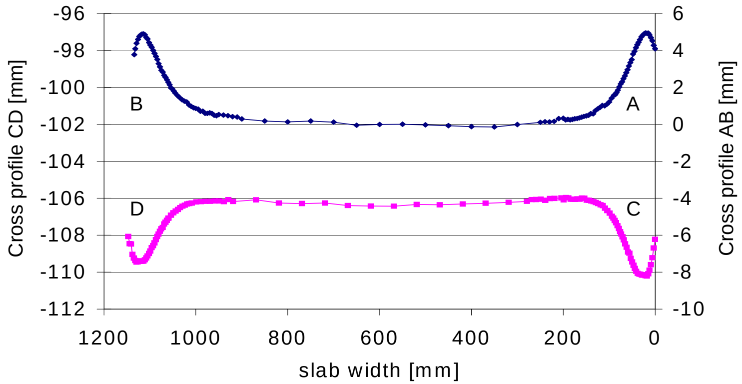
Industrial applications
3D Modeling of steel hot rolling



Industrial applications
3D Modeling of steel hot rolling



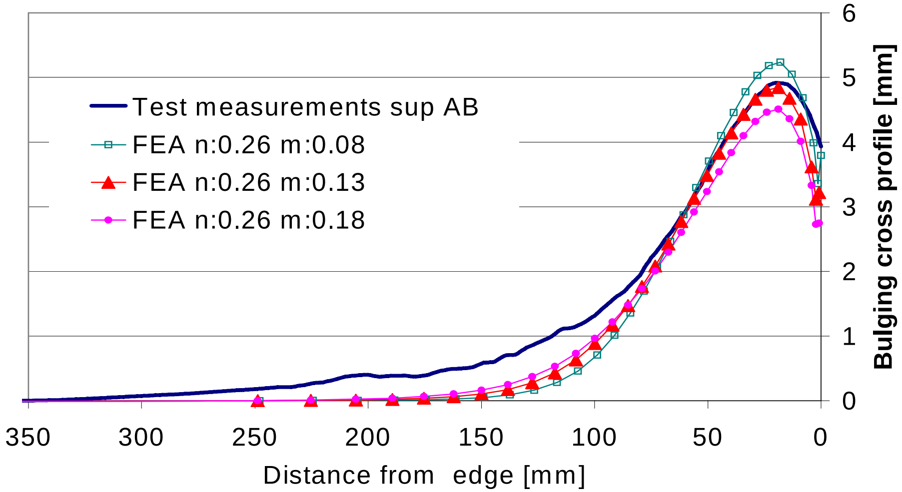




Industrial applications
3D Modeling of steel hot rolling

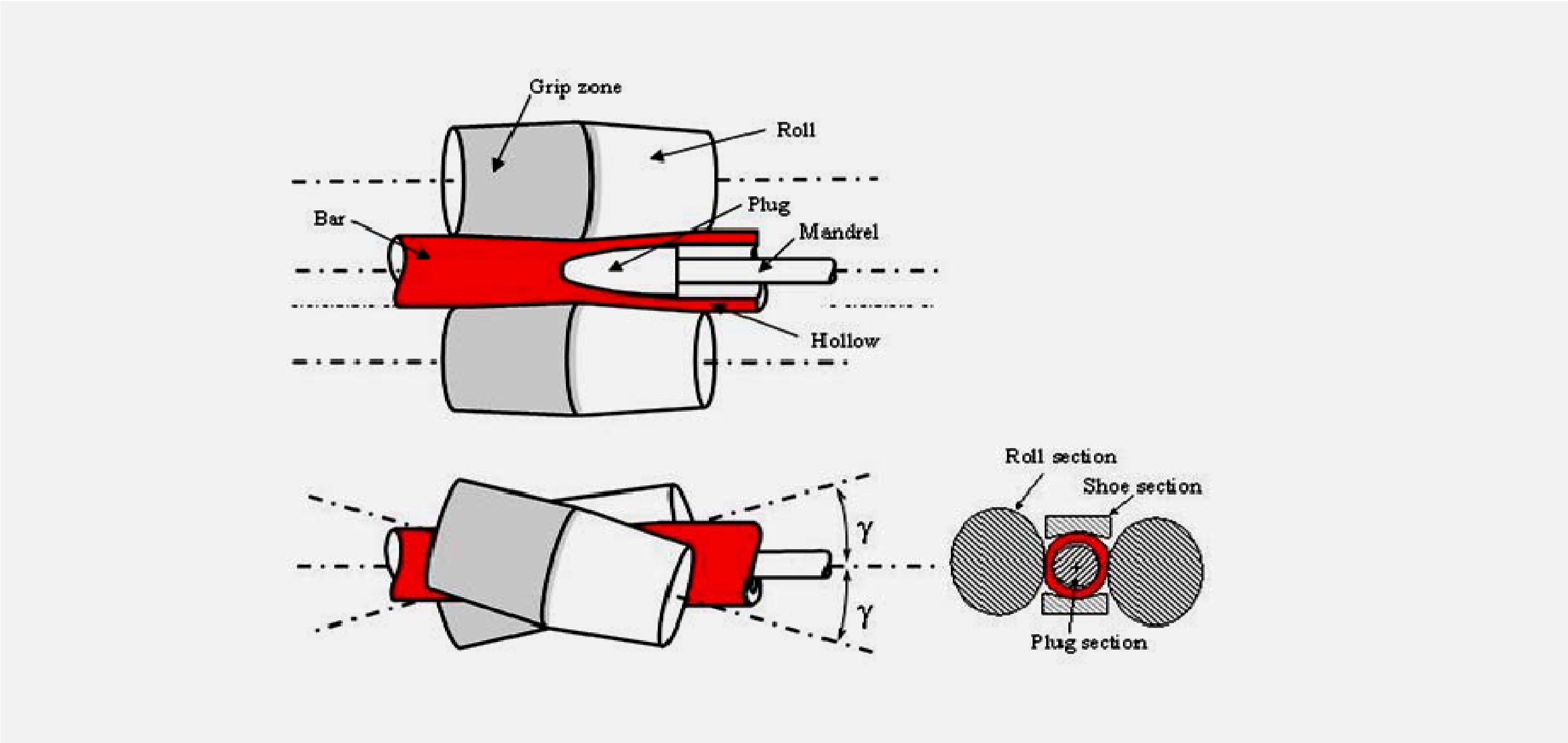
Fields-Backofen constitutive equation

$$\sigma = \sigma_o \varepsilon^n \dot{\varepsilon}^m$$

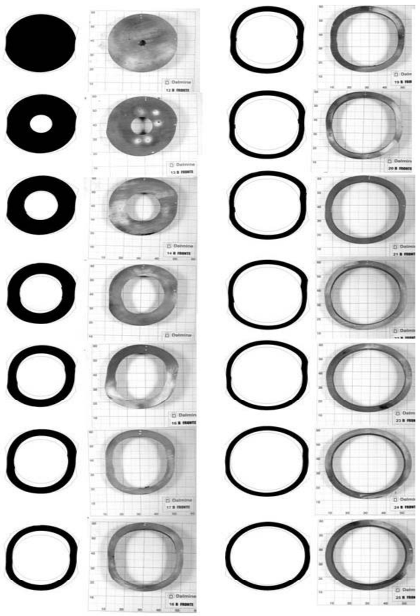


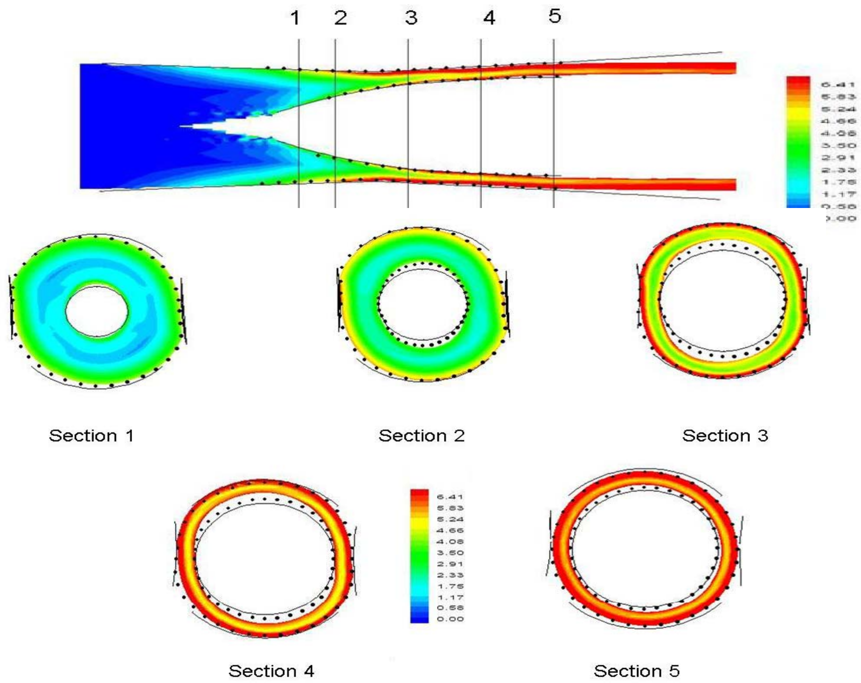
Model Validation

Industrial applications
Modeling the Mannesmann piercing process



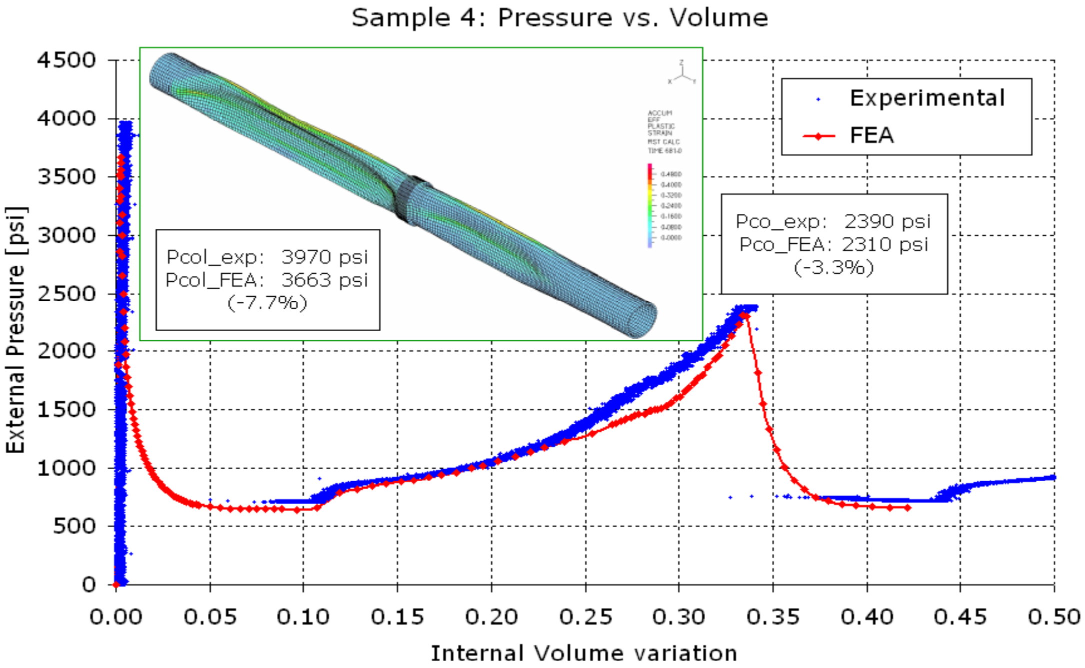
Industrial applications
Modeling the Mannesmann piercing process

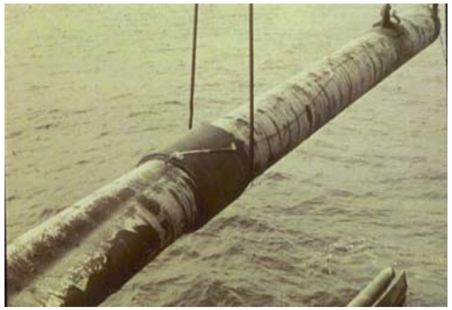




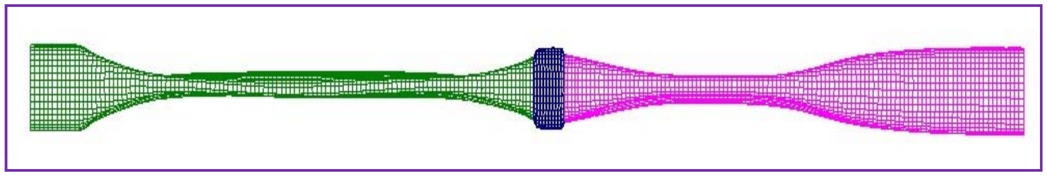
Model Validation

Industrial applications
Marine pipeline collapse

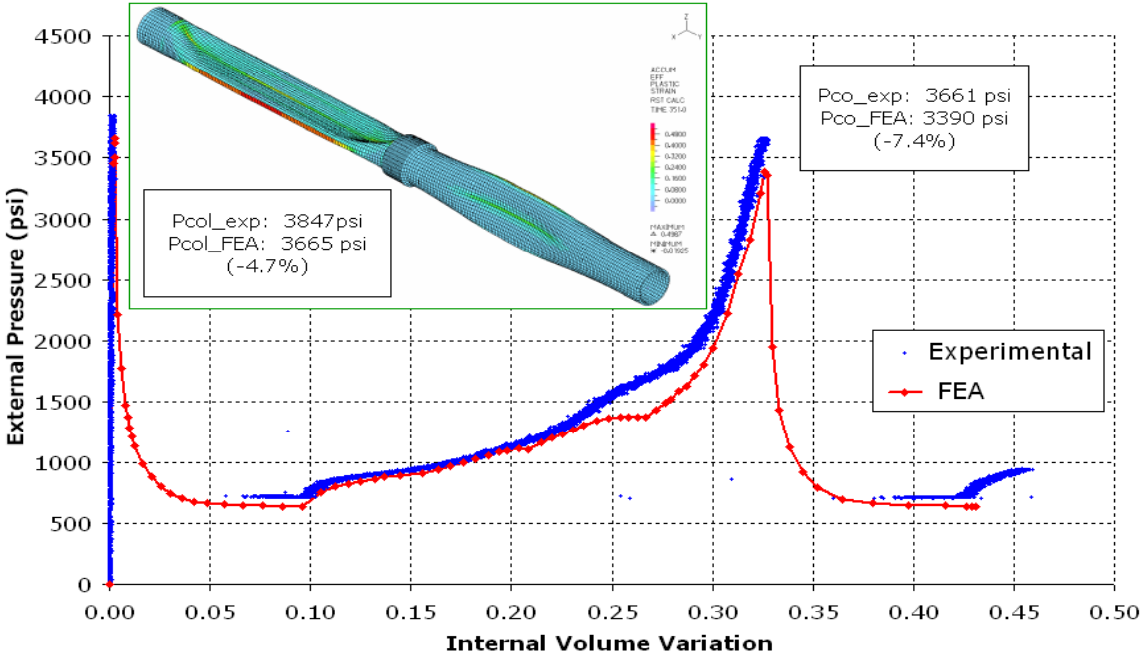




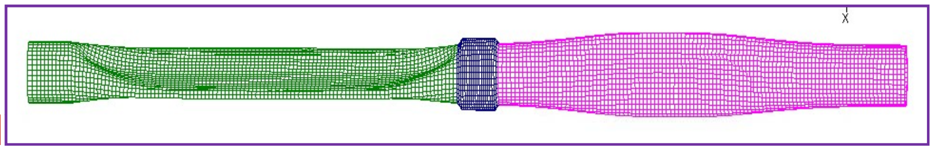
Model Validation



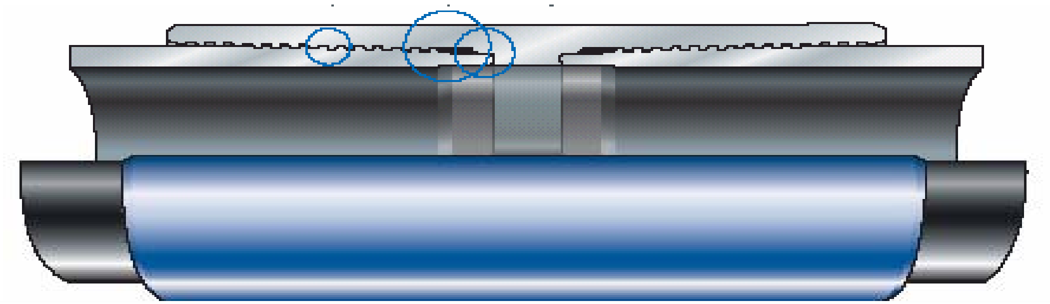
Industrial applications
Marine pipeline collapse

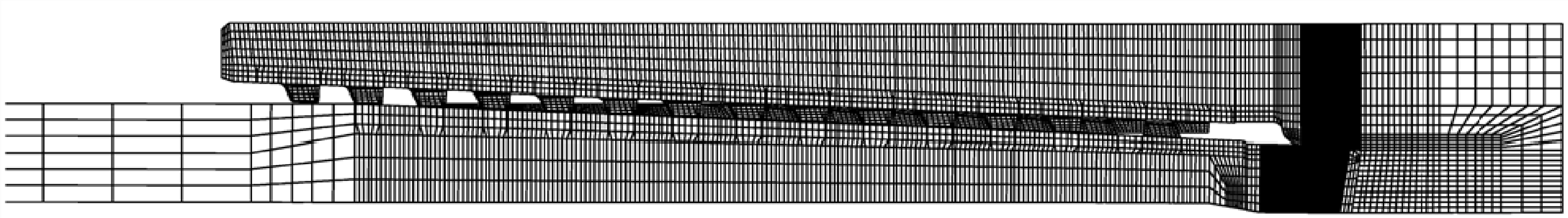


Model Validation

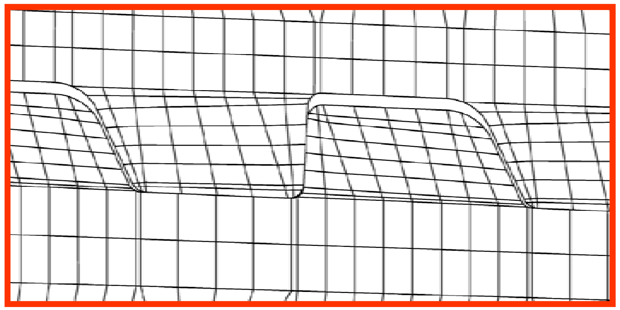


Industrial applications
OCTG Threaded connections

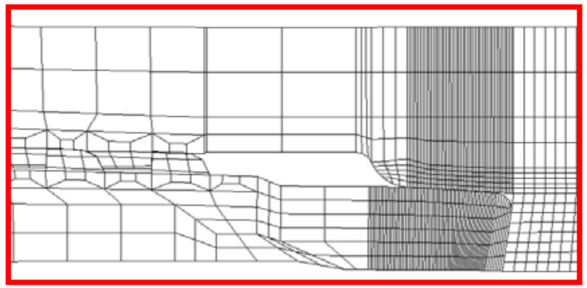




Detail of
the threads

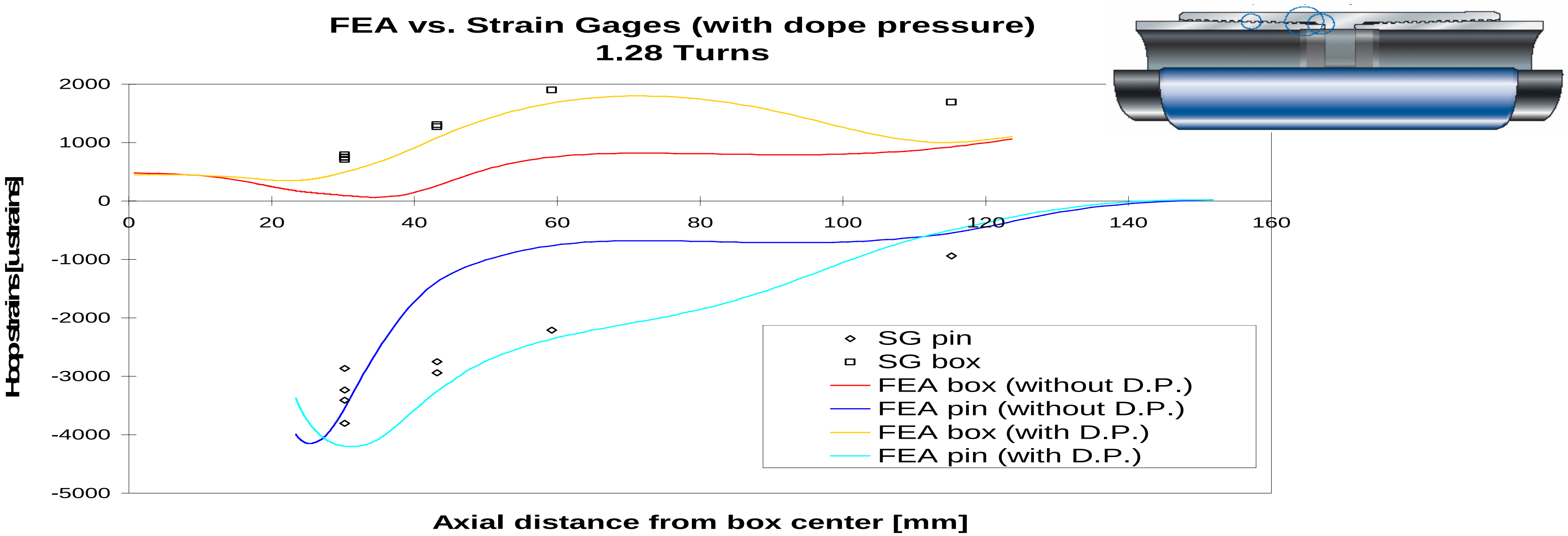


Detail of
the seal area



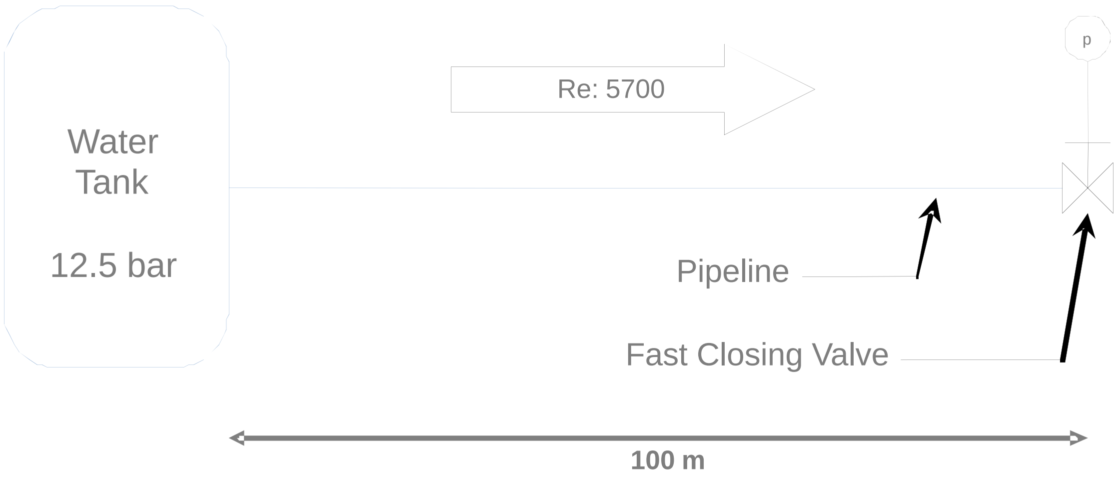
Industrial applications
OCTG Threaded connections

Model Validation



When the dope pressure distribution determined in the full-scale test was included in the finite element model, the numerical results showed a very good agreement with the experimental ones.

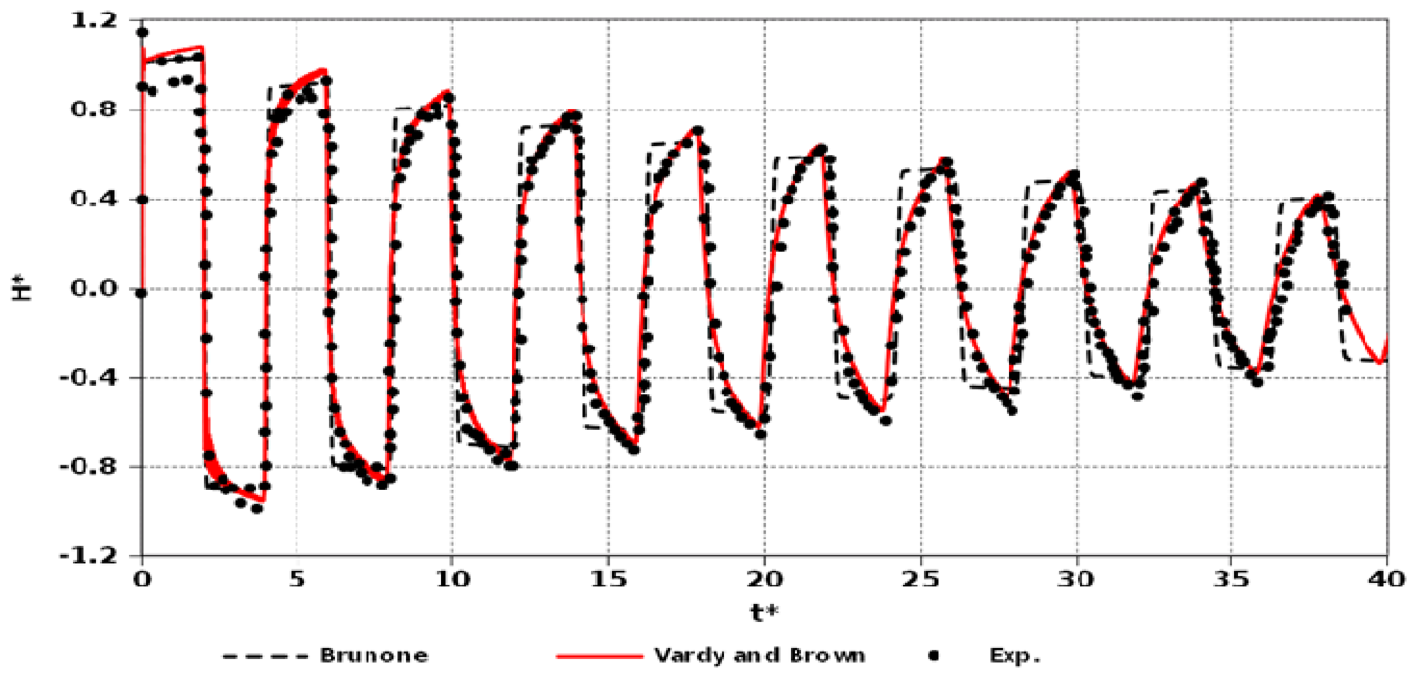
Industrial applications
Waterhammer



Waterhammer experiment. The valve is closed at t=0; the pipe dimensions are L=100m; ID=0.016m and OD=0.018m. Fluid: water

Industrial applications
Waterhammer

Model Validation



Normalized pressure at the valve. Comparison of calculated and experimental results

Nomenclature

Summation convention

$$\sum_{a=1}^{3} v_a r_a = v_a r_a$$

Tensors

Using Cartesian coordinates

Vectors

$$\underline{v} = v_a \underline{e}_a$$

Second order tensors (dyadic representation)

$$\underline{\underline{t}} = t_{ab} \underline{e}_a \underline{e}_b$$

In some references

$$\underline{\underline{t}} = t_{ab} \underline{e}_a \otimes \underline{e}_b$$

Nomenclature

Tensor operations

$$\underline{v} = \underline{c} \cdot \underline{\underline{t}} = c_l \underline{e}_l \cdot t_{ab} \underline{e}_a \underline{e}_b = (c_l t_{ab}) \delta_{la} \underline{e}_b$$
$$= (c_a t_{ab}) \underline{e}_b$$

$$\underline{b} = \underline{\underline{t}} \cdot \underline{c} = t_{ab} \underline{e}_a \underline{e}_b \cdot c_l \underline{e}_l = t_{ab} c_b \underline{e}_a$$

$$\alpha = \underline{\underline{a}} : \underline{\underline{b}} = a_{ij} \underline{e}_i \underline{e}_j : b_{lm} \underline{e}_l \underline{e}_m = (a_{ij} b_{lm}) \delta_{il} \delta_{jm}$$
$$= a_{ij} b_{ij}$$

$$\beta = \underline{\underline{a}} \cdot\cdot\, \underline{\underline{b}} = a_{ij} \underline{e}_i \underline{e}_j \cdot\cdot\, b_{lm} \underline{e}_l \underline{e}_m = (a_{ij} b_{lm}) \delta_{jl} \delta_{im}$$
$$= (a_{ij} b_{ji})$$

Nomenclature

$$\underline{\underline{\underline{\underline{c}}}} = \underline{\underline{a}}\,\underline{\underline{b}} = a_{ij} b_{lm} \underline{e}_i \underline{e}_j \underline{e}_l \underline{e}_m$$