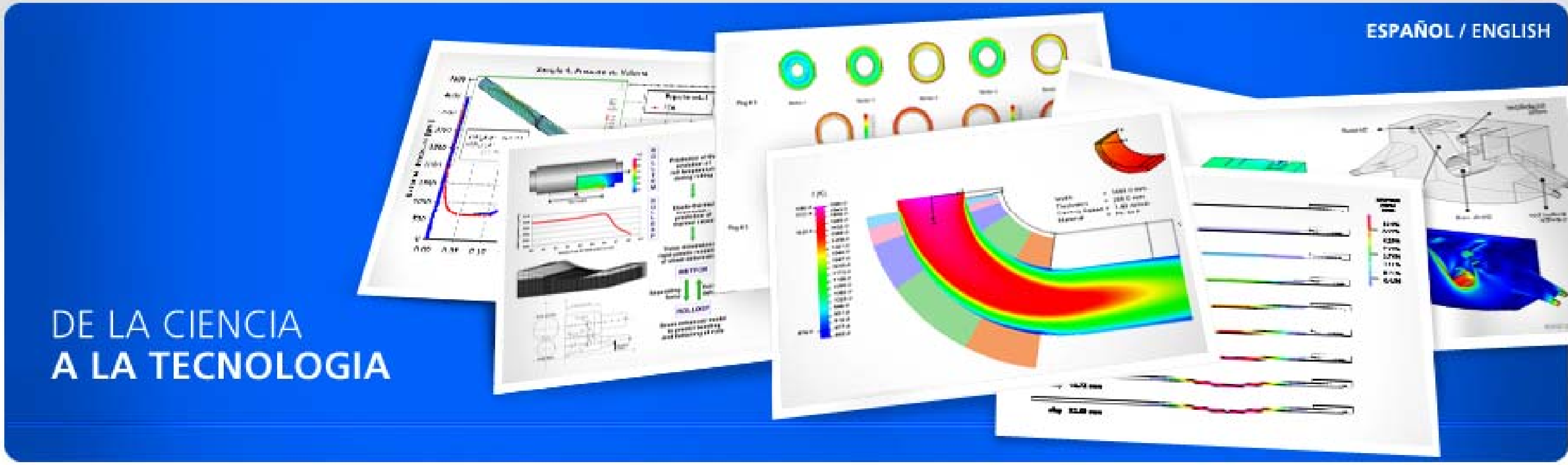
SIM&TEC

Simulación y Tecnología

Simulation and Technology



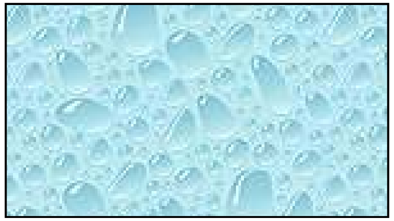
FINITE ELEMENT METHOD IN FLUID DYNAMICS

Part 5: Thermo-fluid dynamics

Marcela B. Goldschmit

Thermo-fluid dynamic model

Walls functions



Boundary layer

Solid

The fluid finite element mesh is located at a wall distance Δ_{wall}.

The friction velocity u^* is calculated solving the nonlinear equation

$$\frac{v_x}{u^*} = \frac{1}{\kappa}\ln\left(\frac{y\,u^*\,E}{\upsilon}\right) \qquad \frac{\rho\,y\,u^*}{\mu} > 11.63$$

The following boundary conditions are applied in the corresponding fluid node.

$$\tau_w = \rho\,u^{*2} \qquad k = \frac{u^{*2}}{\sqrt{C_\mu}} \qquad \varepsilon = \frac{u^{*3}}{\kappa\,y}$$

Boundary layer temperature profile

$$T^+ = \begin{cases} \Pr y^+ & y^+ < y_0^{\theta+} \\ \sigma^\theta\left[\frac{1}{k}\ln y^+ + P_T\right] & y^+ > y_0^{\theta+} \end{cases}$$

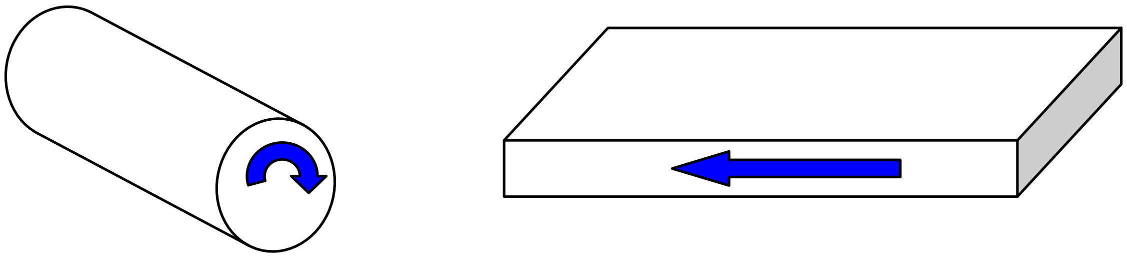
$$y^+ = \frac{\rho\,y\,u_*}{\mu} \qquad P_T = \frac{1}{\sigma^\theta}\left[\Pr y_0^{\theta+} - \frac{\sigma^\theta}{\kappa}\ln y_0^{\theta+}\right]$$

Knowing the solid wall temperature T_s a heat flow is applied

$$Q = h\left(T_s - T_f\right) \qquad h = \frac{\rho C p u_*}{T^+}$$

Thermo-fluid dynamic model

Solid thermal model



$$\rho_s Cp_s \left(\frac{\partial T_s}{\partial t} + \mathbf{v}_s . \nabla T_s \right) - \nabla \cdot \left[\lambda_s \nabla T_s \right] + Q_s = 0$$

The term $\rho_s Cp_s \, \mathbf{v}.\nabla T_s$ allows modeling a moving solid seen from an eulerian point of view as to be rotating cylinders or plates moving in the direction of its axis.

Thermo-fluid dynamic model

Coupling between fluid and solid model

A connection between the solid and the fluid exists due to the tensions in the solid - fluid interface. These tensions are modeled through the wall functions, which were modified to consider the velocity of the moving solid contour, which is always tangent to the fluid - solid interface

$$\frac{v_x - v_s}{u^*} = \frac{1}{\kappa} \ln\left(\frac{y u^* E}{\upsilon}\right)$$

A thermal connection between the fluid and the solid exists due to the heat exchange between both domains through the fluid - solid interface. This heat exchange is modeled by means of a Newton cooling law.

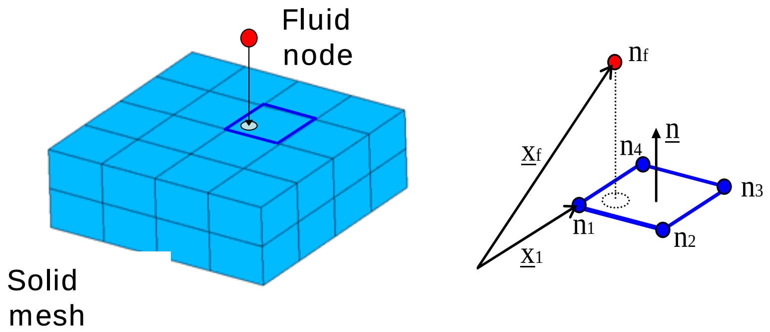
$$Q_f = h(T_s - T_f)$$

$$Q_s = -Q_f = h(T_f - T_s)$$

Thermo-fluid dynamic model

Coupling scheme between different domains

For each contour node of the fluid mesh is necessary to know the solid contour velocity value in that point.



- a superficial element is taken from the solid domain and the distance between the fluid node and the plane that contain the superficial element is calculated.

$$dist = (\underline{x}_f - \underline{x}_1) \bullet \underline{n}$$

- If the *dist* value is negative or is not equal to Δ_{wall}, the element selected is rejected and another element is analyzed.

- If the *dist* value is equal to Δ_{wall}, the intersection point $\underline{x}_i$ is obtained $\underline{x}_i = \underline{x}_f - dist \cdot \underline{n}$

Thermo-fluid dynamic model

Coupling scheme between different domains

The natural coordinates (r_i,s_i) are calculated, using the intersection point position xi and the four nodes coordinates x_1,... x_4, solving the nonlinear equation system

$$\underbrace{\begin{bmatrix} h_1(r_i,s_i)\dots h_4(r_i,s_i) & 0 & 0 \\ 0 & h_1(r_i,s_i)\dots h_4(r_i,s_i) & 0 \\ 0 & 0 & h_1(r_i,s_i)\dots h_4(r_i,s_i) \end{bmatrix}}_{\tilde{\mathbf{H}}} \begin{bmatrix} x_1 \\ \vdots \\ x_4 \\ y_1 \\ \vdots \\ y_4 \\ z_1 \\ \vdots \\ z_4 \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

If r_i and s_i values are between [-1,1] the projection point is the correct. In the opposite case, the element selected is rejected and another surface element is analyzed.

It is possible to obtain for each fluid contour node the corresponding solid velocity $\mathbf{v}_s^f$ according to

$$\tilde{\mathbf{H}}(r_i,s_i)\ \mathbf{V}_s^e = \mathbf{V}_s^f$$

Thermo-fluid dynamic model

Coupling scheme between different domains

The heat exchange term is discretized by finite element method

$$\mathbf{Q}_\mathrm{f} = \left[h \int_{\Omega_e} \mathbf{H}^T \mathbf{H} \, d\Omega_e \right] \cdot \hat{\mathbf{T}}_\mathrm{s}^\mathrm{f} - \left[h \int_{\Omega_e} \mathbf{H}^T \mathbf{H} \, d\Omega_e \right] \cdot \hat{\mathbf{T}}_\mathrm{f}$$

Solid temperature evaluated at the fluid contours nodes

There are two possible solving schemes for the solid – fluid thermal coupling

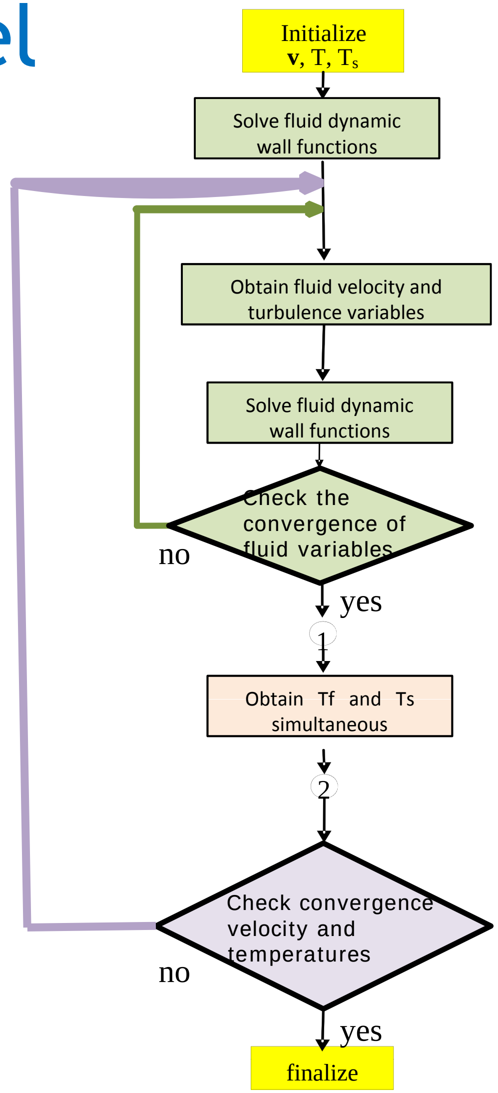
- Segregater: iterative process
- Simultaneous

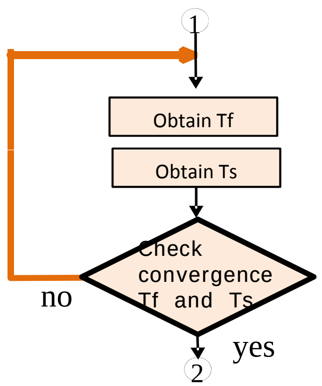
$$\begin{bmatrix} \mathbf{K}_\mathrm{ff} & \mathbf{K}_\mathrm{fs} \\ \mathbf{K}_\mathrm{sf} & \mathbf{K}_\mathrm{ss} \end{bmatrix} \cdot \begin{bmatrix} \hat{\mathbf{T}}_\mathrm{f} \\ \hat{\mathbf{T}}_\mathrm{s} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_\mathrm{f} \\ \mathbf{Q}_\mathrm{s} \end{bmatrix}$$

$$\mathbf{Q}_\mathrm{f}^\mathrm{e} = \underbrace{\left[h \iiint \mathbf{H}^T \mathbf{H} \, d\Omega_e \right] \tilde{\mathbf{H}}(r_i, s_i)}_{\mathbf{K}_\mathrm{fs}} \cdot \hat{\mathbf{T}}_\mathrm{s}^\mathrm{e} - \underbrace{\left[h \iiint \mathbf{H}^T \mathbf{H} \, d\Omega_e \right]}_{\mathbf{K}_\mathrm{ff}} \hat{\mathbf{T}}_\mathrm{f}^\mathrm{e}$$

$$\mathbf{Q}_\mathrm{s}^\mathrm{e} = \underbrace{\left[h \iiint \mathbf{H}^T \mathbf{H} \, d\Omega_e \right] \tilde{\mathbf{H}}(r_i, s_i)}_{\mathbf{K}_\mathrm{sf}} \cdot \hat{\mathbf{T}}_\mathrm{f}^\mathrm{e} - \underbrace{\left[h \iiint \mathbf{H}^T \mathbf{H} \, d\Omega_e \right]}_{\mathbf{K}_\mathrm{ss}} \hat{\mathbf{T}}_\mathrm{s}^\mathrm{e}$$

Numerical model

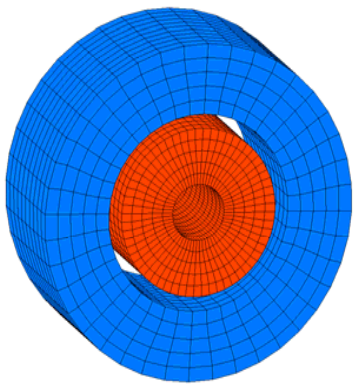




Thermo – fluid dynamic model

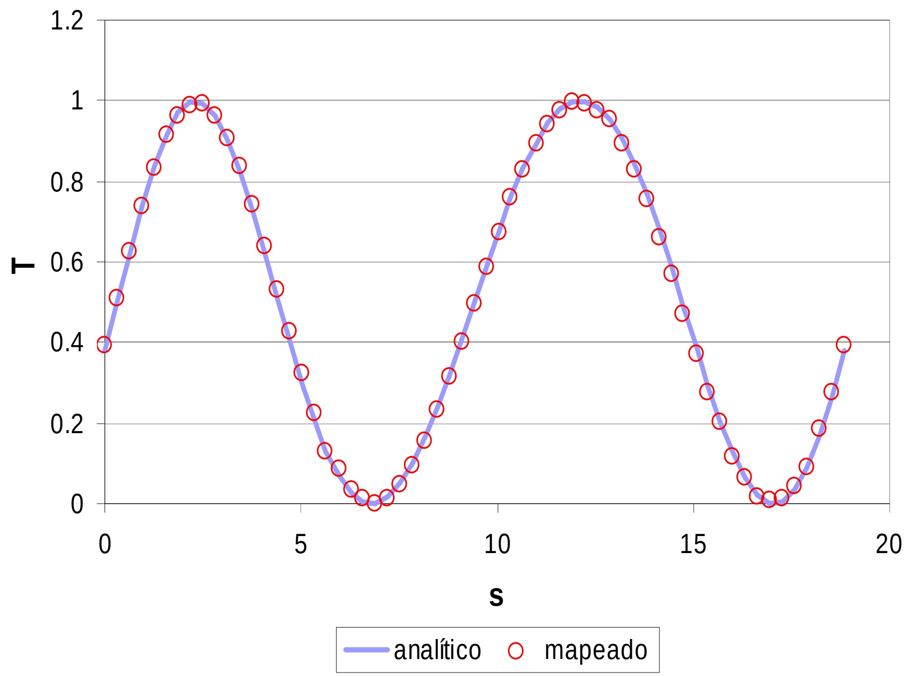
Model verification

The mapping algorithm was tested



The scalar distribution is applied to the internal contour of the external ring, taking the form

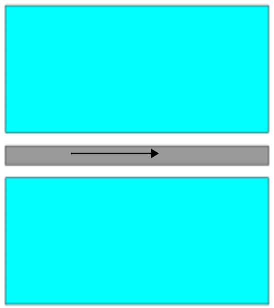
$$A = sen^2(\theta)$$

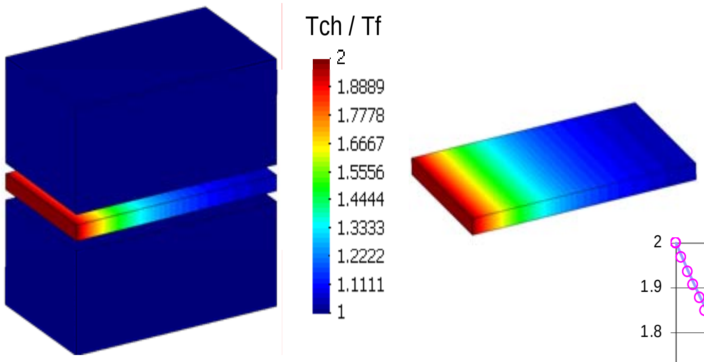


Thermo – fluid dynamic model

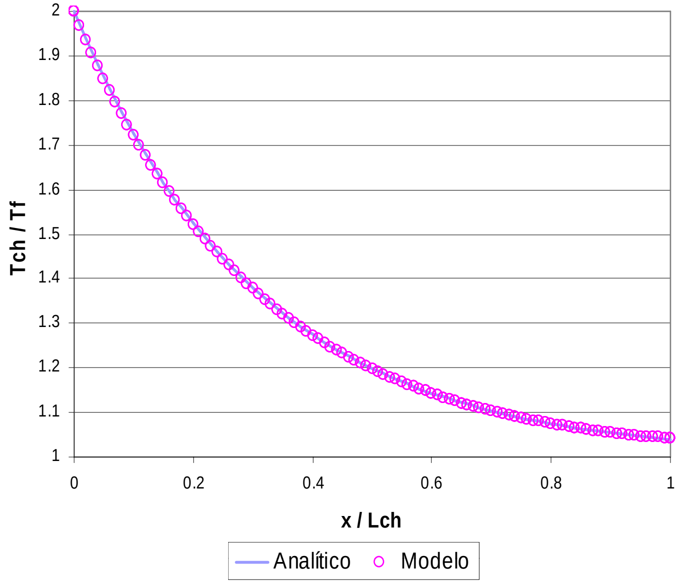
Model verification

The solid energy equation convective term was tested





For the fluid domain the velocity was fixed to zero and the temperature to T_f. On the solid domain, the velocity was fixed to v_s and the inlet temperature was fixed to T_s^i.



Analytical results

$$T_{ch} = (T_i - T_f)\exp\left(\frac{-2h\,x}{\rho e c p v_{ch}}\right) + T_f$$