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Mechanics of Bar Straightening, Part 1: General Analysis of Straightening Process

In the following paper, a general analysis of the mechanics involved in the straightening process of bars and sections by reverse kinematic loading has been made.

Introduction

In re-rolling industries for light bar and sections, the bars and sections are given certain specified degree of straightness [1]. Various types of straightening machines are in use and they may be classified broadly into three groups viz. cross-roll straighteners, section straighteners, and stretch straighteners, based on the basic principles on which they operate. In the first two categories of machines, the bar is made to pass through a set of staggered rolls, producing reverse kinematic loading of the bar, resulting in redistribution of plastic strains, which secures the desired straightening.

Though several varieties of very good straightening machines have been developed by different manufacturers, unfortunately, their design development seems not to be backed up by a detailed theoretical analysis of the plastic deformation processes involved therein. Some analysis of residual stress patterns and power consumption in straightening have been made by White and Briggs [2], Tselivok and Smirnoff [3] and Tokunaga [4]; except for the analysis done by Das Talukder and Johnson [5, 6] and Yu and Johnson [7], no theoretical investigations have been made of the straightening processes and the influences of roll arrangements on the throughput speed and the degree of straightness of the bars produced.

Below, a detailed analysis of the mechanics involved in the straightening processes of the bars; which behave both anisotropically as well as nonhomogeneously, along with the evaluation of degree of straightness of the final product has been made. Further and closer analysis of the processes in different types of machines has been made in Part 2.

General Analysis of Straightening by Reverse Kinematic-Loading (Curvature Change-resisting Moment Curves)

Assuming that the stress-strain curves based on tests by Davis et al [8] as representative curves for ductile behavior of materials, the relationship between stress σ, in reverse loading and change in longitudinal strain, ϵ [Fig. 1(*a*)], may be shown by a family of curves shown in Fig. 1(*b*), for different initial residual strains ϵ_r. The results reported in [8] show that the defined yield point is absent during reverse loading and instead curves are sharply curved near that region. In the following it is assumed that:

(*a*) Yield point stress for an initially unstrained bar is the same in tension and compression.

(*b*) Stress-strain relation in reverse loading is bilineal, which is very close to the results reported in [8]. The assumed bilineal relationship may be expressed as

$$\begin{aligned} \sigma &= E\epsilon & \epsilon \leq (\epsilon_y + c\epsilon_r) \\ &= E[\epsilon_y + c\epsilon_r] + E' \ [\epsilon - (\epsilon_y + c\epsilon_r)] & \epsilon \geq (\epsilon_y + c\epsilon_r) \end{aligned} \quad (1)$$

where, E and E' are elastic and plastic moduli, respectively, ϵ_y is the yield strain in reverse loading for an element with no initial residual strain, and c is a constant and $c \ll 1$.

Let a bar with a section, symmetrical about two mutually perpendicular axes, y and z with initial residual curvature k_r in one of the plane of symmetry be considered.

Since the bar has an initial curvature k_r, the initial residual strains in the fibers at different distances from the z-axis will be different. Hence for different fibers strain change-stress curves will be different as shown in Fig. 1(*b*) corresponding to a particular strain change, stress must be found out from the curve of the initial residual strain the particular fiber has. If the residual strain in the extreme fiber is ϵ_{r1}, and the residual strain in the fiber at a distance y from the z-axis is ϵ_r, then

$$\epsilon_{r_1}/(h/2) = \epsilon_r/y = k_r \quad (2)$$

where h is the depth of the section considered. If, due to the straightening, the change in curvature in the bar length is k, then from Fig. 2:

$$\begin{aligned} k &= 2\epsilon_1/h \\ \text{and } \epsilon &= 2\epsilon_1 y/h \end{aligned} \quad (3)$$

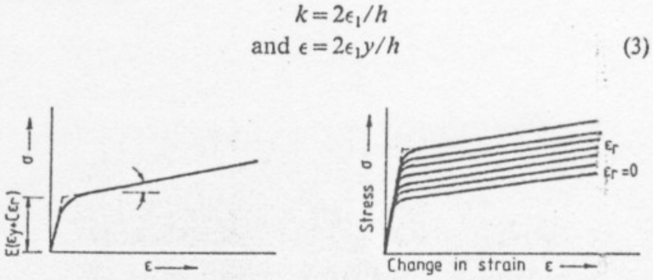

Fig. 1 Stress-strain change curves for different initial residual strains

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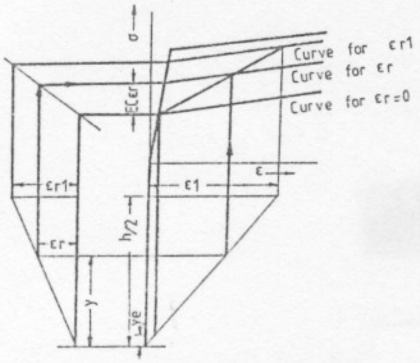

Fig. 2 Variation of stress across the depth in bending

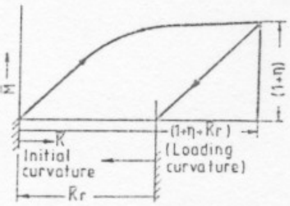

Fig. 3 Initial curvature and loading moment

where, ϵ and ϵ_1 are the change in strain in a fiber at y and $h/2$, respectively.

If y_e is the value of y at elastic-plastic boundary, then from equation (3)

$$\begin{aligned} \sigma &= Eky \qquad\qquad , -y_e \le y \le y_e \\ &= [E-E']\epsilon_y + \xi y \quad , \ y_e \le |y| \le h/2 \qquad (4) \end{aligned}$$

where $\xi = [E-E']ck_r + E'k$

So, the moment M, required to produce the change is given by

$$\begin{aligned} M &= [\lambda EK + (1-\lambda)\xi]I_z, \ k \ge (k_y + ck_r) \\ & \qquad\qquad 0 \le \lambda \le 1 \qquad (5) \\ &= E\, I_z k \qquad\qquad , \ k \le (k_y + ck_r) \end{aligned}$$

where λ is a parameter dependent on the section parameters and y_e.

But

$$y_e = \epsilon_y / h \qquad (6)$$

So, λ is a function of the section parameters and the change in curvature.

If the moment required to produce the same curvature change k in a bar length, having zero initial curvature is represented by $M_0(k)$, then from equations (4) and (5)

$$M_0 = [E + (1-\lambda)E']kI_z \qquad (7)$$

from equations (5) and (7)

$$\Delta M = M - M_0 = [1-\lambda][E-E']\ ck_r I_z \qquad (8)$$

where M is the extra moment required to produce a particular curvature, due to initial curvature in the bar. Writing

$$M/M_y = \overline{M},\ E'/E = \overline{E}',\ k/k_y = \overline{k}$$

where M_y and k_y are yield moment and yield curvature of an initially unstrained bar and $M_y = EI_z k_y$. Equations (5) and (8) may be expressed as

$$\begin{aligned} \overline{M} &= \overline{k} \qquad\qquad \overline{k} \le (1 + c\overline{k}_r) \\ &= \lambda \overline{k} + (1-\lambda)\overline{\xi} \qquad \overline{k} \ge (1 + c\overline{k}_r) \qquad (9) \\ \Delta \overline{M} &= c[1-\overline{E}'][1-\lambda]\overline{k}_r \end{aligned}$$

where, $\overline{\xi} = \xi/(Ek_y) = c[1-\overline{E}']\overline{k}_r + \overline{E}'\overline{k}$

Since the unloading from the reverse loading is also elastic, i.e.,

$$\frac{d\overline{M}}{d\overline{k}} = 1, \quad \overline{M}d\overline{M} < 0 \qquad (10)$$

the moment, which will have to be applied to produce a curvature change $\overline{k}_r$ in a bar with initial curvature $\overline{k}_r$ may be obtained by putting

$$\overline{M} = \overline{k} - \overline{k}_r$$

in equation (9) (Fig. 3); let the corresponding value of the moment be $\overline{M}_1$ and it may be expressed as

$$\overline{M}_1 = 1 + \eta(\overline{k}_r) \qquad (11)$$

Evidently

$$\eta(0) = 0 \qquad (12)$$

The corresponding curvature change $\overline{k}_1$ is given by

$$\overline{k}_1 = 1 + \eta + \overline{k}_r \qquad (13)$$

From equations (9), (10), and (13)

$$\overline{k}_1 = [c + \{(1-\lambda)\ (1-\overline{E}')\}^{-1}]\overline{k}_r \qquad (14)$$

So, from equations (13) and (14)

$$\eta = [\{(1-\lambda)\ (1-\overline{E}')\}^{-1} - (1-c)]\overline{k}_r \qquad (15)$$

But λ is a function of $\overline{k}$ and so of $\overline{k}_1$ and $\overline{k}_1 \ge \overline{k}_r$. For example for a circular section

$$1-\lambda = 1 - \frac{2}{\pi}\sin^{-1}(1/\overline{k}) + (\overline{k}^2-1)^{1/2}(\overline{k}^2-2)/\overline{k}^4 \qquad (16)$$

The variation of $(1-\lambda)$ with $\overline{k}_1$, for a circular section, is shown in Fig. 4. It can be observed from equation (16) that for circular sections

$$1-\lambda(1) = 0 \text{ and } 1-\lambda(\infty) = 1$$

Nomenclature

c = a constant
E = elastic modulus for the material of the bar
E' = plastic modulus for the material of the bar
h = depth of a section of the bar
k = change in curvature in the bar length
k_r = initial residual curvature
k_f = final residual curvature
k_n = highest value of k_f due to variation in M_1
k_p = highest value k_f due to early unloading
M = bending moment required to produce curvature change k
M_1 = loading moment (at mid-span)
M_0 = bending moment required to produce curvature change k in a bar with no initial curvature
m_h = highest moment developed in the plane of initial curvature
y_e = value of y at elastic-plastic boundary
ϵ = change in strain
ϵ_r = initial residual strain
ϵ_r^* = particular value of ϵ_r
ϵ_y = yield strain in tension or compression
σ = normal stress
η = a function of curvature
η^* = a function of curvature for particular value of k_r
λ = a dimensionless parameter varying between 0 and 1
ζ = a dimensionless parameter ≤ 1
θ = angle between the plane of loading and plane of initial curvature
ξ = a parameter, defined in the text

All variables with a bar over them are nondimensionalized variables defined in the text.

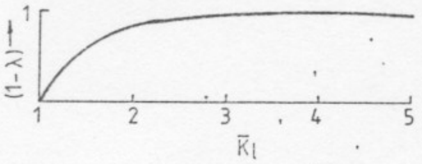

Fig. 4

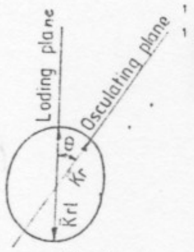

Fig. 5

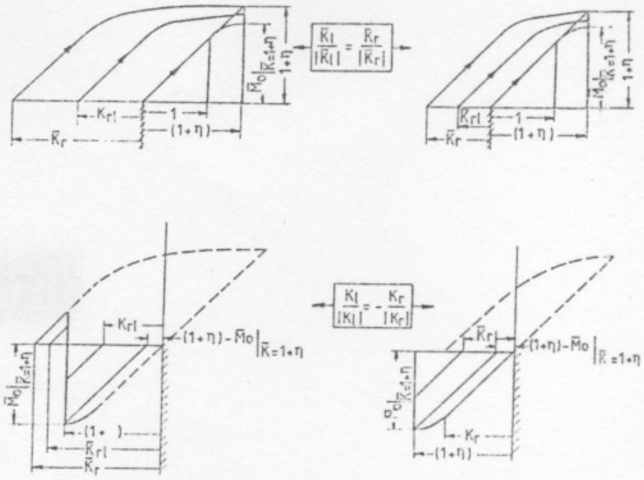

Fig. 6 Effect of variation of the angle between the loading plane and osculating plane on the loading moment developed

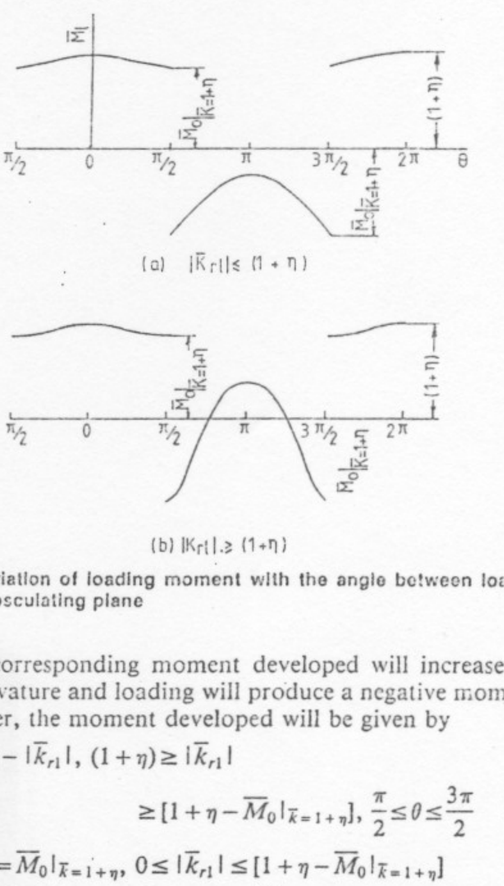

Fig. 7 Variation of loading moment with the angle between loading plane and osculating plane

and the value of λ quickly approaches 1. Again, since $\overline{E}' \ll 1$ from equation (15), it can be seen that unless $\bar{k}_1$ is near about 1 i.e., for higher values of $\bar{k}_1$

$$\eta \ll \bar{k}_r \tag{17}$$

Loading for Straightening

If a length of a bar has an initial residual curvature $\bar{k}_r$, then to straighten it, i.e., to have final residual curvature equal to zero, a loading moment in reverse direction (assumed positive) must be applied to have an additional residual curvature $\bar{k}_r$ in the reverse direction (assumed positive), so that the resulting curvature is zero. It can be seen from Fig. 3, that a kinematic loading with curvature given by equation (13) will bring the final residual curvature to zero value. So, the straightening process basically involves the application of a kinematic loading given by equation (13) to the section involved.

In the following, unless otherwise mentioned the plane of loading will always be assumed to be vertical for convenience of analysis and a positive change in curvature (loosely mentioned as "curvature") will always mean that it is opposite in nature to the initial curvature of the bar length to be straightened. Let the principal plane of initial curvature (osculating plane) make an angle θ with the plane of loading. If $\bar{k}_r$ is the initial curvature in the osculating plane, then initial curvature $\bar{k}_{r1}$ in the plane of loading is (Fig. 5)

$$\bar{k}_{r1} = \bar{k}_r \cos\theta \tag{18}$$

So, the change in curvature due to kinematic loading is (Fig. 6)

$$\bar{k}_1 = 1 + \eta + \bar{k}_r \cos\theta \tag{19}$$

If $\bar{k}_{r1} \ll (1+\eta)$, then for values of θ in the range $-\pi/2 \leq \theta \leq \pi/2$, $\bar{k}_{r1}$ has the opposite sign of $\bar{k}_1$; so the change of curvature is

$$\bar{k}_1 = [1 + \eta + |\bar{k}_{r1}|] \geq (1+\eta) > 0, \quad \frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2} \tag{20}$$

The corresponding moment developed at the section considered will be a positive loading moment given by

$$\overline{M}_1 = \overline{M}\,(\bar{k}, \bar{k}_{r1})|_{\bar{k}=1+\eta+\bar{k}_{r1}}, \quad \frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2} \tag{21}$$

where $\overline{M}(\bar{k}, \bar{k}_{r1})$ represents the function relating resisting moment, change in curvature $\bar{k}$, for initial curvature $\bar{k}_{r1}$. If θ is the range (20), then $\bar{k}_{r1}$ has the same sign as $\bar{k}_1$ and so

$$\bar{k}_1 = 1 + \eta - |\bar{k}_{r1}|, \quad \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \tag{22}$$

and the corresponding moment developed will increase the initial curvature and loading will produce a negative moment.

However, the moment developed will be given by

$$\overline{M}_1 = 1 + \eta - |\bar{k}_{r1}|, \; (1+\eta) \geq |\bar{k}_{r1}|$$

$$\geq [1 + \eta - \overline{M}_0|_{\bar{k}=1+\eta}], \; \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \tag{23}$$

$$= \overline{M}_0|_{\bar{k}=1+\eta}, \; 0 \leq |\bar{k}_{r1}| \leq [1 + \eta - \overline{M}_0|_{\bar{k}=1+\eta}]$$

which is fully elastic. If however, $\bar{k}_{r1} \geq (1+\eta)$, then

$$\overline{M}_1 = \overline{M}(\bar{k}, \bar{k}_{r1})|_{\bar{k}=1+\eta+\bar{k}_{r1}}, \quad \frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2} \tag{24}$$

$$\overline{M}_1 = \overline{M}(\bar{k}, \bar{k}_{r1})|_{\bar{k}=|\bar{k}_{r1}|-1-\eta}, \quad \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \tag{25}$$

and $\overline{M}_1 < \overline{M}(\bar{k}, \bar{k}_r)|_{\bar{k}=\bar{k}_r}$

The nature of variation of $\overline{M}_1$ with the value of 0 has been shown in Fig. 7, for $|\bar{k}_{r1}| \leq (1+\eta)$ and $|\bar{k}_{r1}| \geq (1+\eta)$. Thus,

$$\overline{M}_1 = \overline{M}_1\,(\bar{k}_r, \theta) \tag{26}$$

If a length of a bar is moved through a system of staggered rolls in a straightening machine, then except for the end rolls, which may be considered as "support rolls" for the beam length of the bar, all other rolls will apply kinematic loading

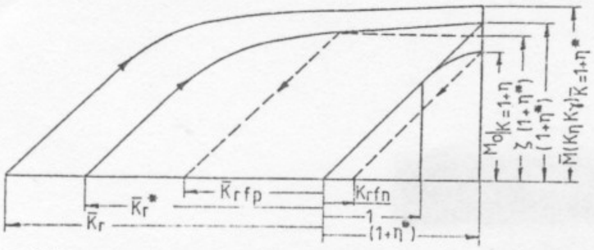

Fig. 8 Effect of variation in initial curvature on the loading moment developed

on the beam, due to their displaced position from the line of support rolls. If the rolls are so placed that a bar with initial curvature $\bar{k}_r^*$, when put through the rolls, develops a loading moment

$$\bar{M}_1 = 1 + \eta^*(\bar{k}_r^*) = 1 + \eta^*, \ \theta = 0 \tag{27}$$

at the sections at the loading roll, then as explained above, the rolls will cause the sections at contact to develop moments (Fig. 8) in the range (nonhomogenity effect)

$$\begin{aligned} &\bar{M}_0|_{(\bar{k}=1+\eta^*)} \le \bar{M}_1 \le (1+\eta^*) &&, \bar{k}_r \le \bar{k}_r^* \\ &\bar{M}_0|_{(\bar{k}=1+\eta^*)} \le \bar{M}_1 \le \bar{M}(\bar{k},\bar{k}_r)|_{\bar{k}=1+\eta^*+\bar{k}_r} &&, \bar{k}_r \le \bar{k}_r^* \end{aligned} \tag{28}$$

depending on the values of $\bar{k}_r$ and θ.

Final Residual Curvature

Let a length of a bar to be straightened, with initial residual curvature $\bar{k}_r$, be passed through the rolls, which have been so adjusted that if the bar length has $\bar{k}_r = \bar{k}_r^*$, then on unloading from the highest moment (with $\theta = 0$), $1+\eta^*$, the change in residual curvature is k_r^*. Since the curvature $\bar{k}_r$ and angle θ may have any value, the highest loading moment experienced by any section of the bar length will be in either of the ranges (30), depending on the relative values of $\bar{k}_r$ and $\bar{k}_r^*$.

If, however, unloading takes place before the section can attain the moment $\bar{M}_1$, say unloading takes place when, (pitch effect)

$$\bar{m}_h = \xi \bar{M}_1 \ , \quad \xi \ll 1 \tag{29}$$

where, $\bar{m}_h$ is the highest value of the component of the loading moment in the plane of curvature at the section, then from (28),

$$\begin{aligned} &\xi\bar{M}_0|_{\bar{k}=1+\eta^*} \le \bar{m}_h \le (1+\eta^*) && \bar{k}_r \le \bar{k}_r^* \\ &\xi\bar{M}_0|_{\bar{k}=1+\eta^*} \le \bar{m}_h \le \bar{M}(\bar{k},\bar{k}_r)|_{\bar{k}=1+\eta^*+\bar{k}_r}, && \bar{k}_r \ge \bar{k}_r^* \end{aligned} \tag{30}$$

If $\bar{k}_n$ (≤ 0) and $\bar{k}_p$ (≥ 0) are largest values of final residual curvatures for variation in loading moment $\bar{M}_1$ and for early unloading, respectively, (Fig. 8), then

$$0 \le |\bar{k}_f| \le |\bar{k}_f^*| \tag{31}$$

where, $\bar{k}_f$ is the final residual curvature and $\bar{k}_f^*$ equals the magnitude of the two final residual curvatures $\bar{k}_n$ and $\bar{k}_p$. Again from equation (9)

$$\frac{d\bar{M}}{d\bar{k}} = \lambda[1-\bar{E}'] + \bar{E}' + (1-\bar{E}')\,[\bar{k} - c\bar{k}_r]\,\frac{d\lambda}{d\bar{k}} < 1 \tag{32}$$

So, for the same variation in $\bar{M}_1$ and $\bar{m}_h$

$$|\bar{k}_n| < |\bar{k}_r| \tag{33}$$

For illustration, if

$$\bar{M}_0|_{\bar{k}=1+\eta} = \xi(1+\eta) = (\bar{m}_h)_{\text{lowest}} \tag{34}$$

$$|\bar{k}_p| = (1+\xi)(1+\eta) / \left(\frac{d\bar{M}}{d\bar{k}}\right)_{\bar{k}=1+\eta+\bar{k}_r} - (1-\xi)(1+\eta) \tag{35}$$

and

$$|\bar{k}_n| = (1-\xi)(1+\eta) \tag{36}$$

From equations (35) and (36)

$$|\bar{k}_p| / |\bar{k}_n| = 1 / \left(\frac{d\bar{M}}{d\bar{k}}\right)_{\bar{k}=1+\eta+\bar{k}_r} - 1 \tag{37}$$

So, unless the initial curvature is small compared to the curvature of roll setting, i.e., unless

$$\begin{aligned} &\bar{k}_r/(1+\eta+\bar{k}_r) \ll 1 \\ &|\bar{k}_p| > |\bar{k}_n| \, . \end{aligned} \tag{38}$$

Discussion and Conclusion

In the above, the mechanics of straightening of bars and sections with initial residual curvature distributed nonhomogeneously along the length of the bar by reverse kinematic loading has been dealt with graph-analytically in details in general terms. If the range of initial residual curvature of the bars to be straightened is known, then for a particular set-up of kinematic loading, the range of final residual curvature, i.e., the degree of straightness of the product, can be ascertained. Conversely, if the desired degree of straightness is specified the required loading may be ascertained. The analysis, as will be shown later in Part 2 of the article where the general analysis is particularized for different types of straighteners, will be useful in bringing out clearly the relative merits and demerits of different types of straighteners and the scope for further improvement in the design of straighteners. However, from the above, the amount of lateral stagger the rolls have to be provided for a particular loading cannot be ascertained and further analytical investigations should be done to determine the same.

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Mechanics of Bar Straightening, Part 2: Straightening in Cross-Roll Straighteners

An analysis is made of the straightening process of rolled circular bars and tubes, with initial residual curvature in different types of cross-roll straighteners indicating the degree of straightness and throughput speed obtainable from them. Suggestions are made for roll arrangement for higher throughput speed and degree of straightness of the product.

Introduction

Light bars and sections are taken from rolling mills slightly bent or distorted and straightened to specified degree in the rerolling industries. This is accomplished for circular bars and tubes in cross-roll straighteners. In these machines the bars and tubes rotate about their own axis as they progress forward through the laterally staggered rolls being subjected to bending beyond elastic limit. These machines, with various combination of driven and idle rolls, are of four basic types [1,2,3] as follows:

(*i*) *Reeling machines or two-roll straighteners:*

In these machines, the bar is bent across the horns of a concave roll (Fig. 1). These machines can further be subdivided into following two types:
(*a*) air-bend, or free-bend straighteners, and
(*b*) line-contact roll straighteners
depending on the profile of the work-rolls. With line-contact rolls the bar length between the rolls or the "beam length" may be considered to be loaded uniformly across the span and with air-bend rolls, the beam length may be considered to be subjected to a concentrated load at mid-span.

(*ii*) *Six-roll straighteners:*

These machines have six rolls mounted in three pairs (Fig. 2). Beam length in these straighteners, which is higher than that in air-bend straighteners may be considered to be subjected to a distributed loading over a length at mid-span.

(*iii*) *Multi-staggered roll straighteners:*

These machines have five, seven, or even ten staggered rolls, such that there are several "loading bays" in each of which the bar is subjected to a concentrated load, consecutive bays being loaded in opposite directions.

(*iv*) *Cluster-roll straighteners:*

These straighteners have seven rolls arranged in two clusters of three rolls with a central deflecting roll. In these machines also the beam length may be considered to be loaded by a concentrated load at the mid-span.

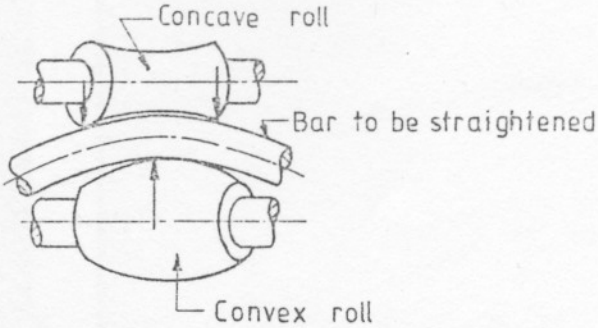

Fig. 1 Bar straightening in a two-roll "air-bend" machine

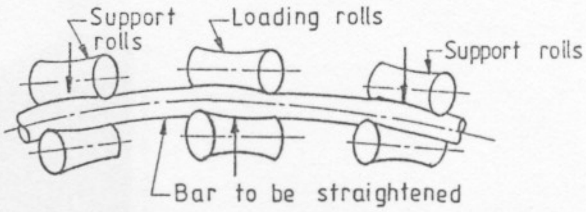

Fig. 2 Bar straightening in a six-roll straightening machine

Though several varieties of very good straightening machines have been developed by different manufacturers, unfortunately their design development seems not to be backed up by a detailed theoretical analysis of the plastic deformation process involved therein. Except for the analysis made by Das Talukder and Johnson [3,4] and Yu and Johnson [5], no theoretical investigation has been made of the straightening processes and the influence of roll arrangements on the throughput speed and the degree of straightness of the product.

Below a detailed analysis of the straightening processes of the initially bent bar in cross-roll straightening machines has been made. Longitudinal fibers of the bar, with different loading history, have been assumed to

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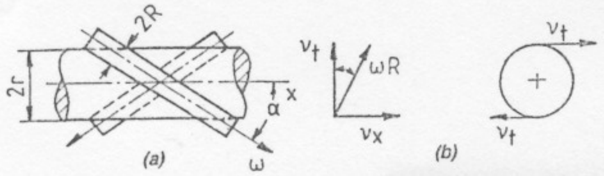


Fig. 3 Motion of the bar through cross-rolls

have different stress-strain relationship and nonhomogeneity of the initial curvature has been taken into consideration while analyzing. Suggestions have been made for further design improvements also.

Loading of the Bar

In a cross-roll straightener, the rolls are set at angles $\pm \alpha$ with the axis of the bar [Fig. 3(*a*)] and cause the bar to rotate about its own axis as it progresses forward through the rolls. If ω is the angular velocity of the rolls of radius R and v_x and v_t are the throughput velocity and tangential velocity of a point on the surface of the bar, then the helix angle in space of the locus of the point of contact is also α [3], i.e.,

$$v_x/v_t = \tan\alpha \tag{1}$$

So, pitch p of the helix is given by

$$p = 2\pi r \tan\alpha$$

$$\text{or } \bar p = p/l = 2\pi \bar r \tan\alpha \tag{2}$$

where l is the beam length, r is the radius of the bar and $\bar r = r/l$. So,

$$\bar v_x = \frac{v_x}{\omega R} = \left[1 + \frac{(2\pi \bar r)^2}{\bar p^2}\right]^{-\frac{1}{2}} \tag{3}$$

Let the plane of initial curvature at a generic section of the beam a distance $\bar x = x/l$ from the support roll at entry make an angle θ with the plane of kinematic loading. Also, let the loading moment in the plane of loading at the generic section be $\bar M = \bar M(\bar x) = \frac{M(x)}{M_y}$ where M_y is the yield point moment for the section. The component of M, which will act in the plane of initial curvature, so far as the straightening process is concerned, is actually the effective bending moment at the section. Thus the effective bending moment $\bar m_\theta$ at the section is

$$\bar m_\theta = \bar M \cos\theta \tag{4}$$

The component $\bar M \sin\theta$ acts in the plane of zero initial curvature and mostly produces elastic deformation. If it produces any plastic deformation, it will be removed or reduced during the passage of the section through effective moment. So this component of $\bar M$ is of no consequence for either analytical or practical purpose. Since

$$\cos\theta = \bar y/\bar r \tag{5}$$

from equation (4) $\bar m_\theta = \bar M \, \bar y/\bar r$

Let the generic section at the entry to the bay i.e., at the first support roll make an angle θ_0 with the plane of loading. Then

$$\bar x = [\omega \bar R \sin\alpha] t \tag{6}$$

$$\text{and } \bar y = \bar r \sin\left[\frac{(\omega \bar R \cos\alpha)t}{\bar r} + \theta_0\right] \tag{7}$$

From equations (5), (6), and (7)

$$\bar m_\theta = \bar M \sin\left\{\frac{2\pi \bar x}{\bar p} + \theta_0\right\} \tag{8}$$

The above equation shows that $\bar m_\theta$ is an oscillating function of $\bar x$ with $\bar M$, as the moment amplitude varying with $\bar x$ (Fig. 4). If $\bar m_\theta$ reaches its first peak value at $\bar x = \bar x^*$, then it will attain peak values $\bar m_p$ at

$$\bar x = \bar x^* + s\bar p \tag{9}$$

in which s takes up the values 1,2,3 etc. If

$$[n+2]\,\bar p \ge \frac{1}{2} \ge [n+1]\,\bar p$$

then $\bar m_p$ will attain its highest value $\bar m_h$ either at (pitch effect)

$$\bar x = \frac{1}{2} - \upsilon \bar p \qquad 0 \le \upsilon \le 1 \tag{10}$$

$$\text{or at } \bar x = \frac{1}{2} + (1-\upsilon)\,\bar p$$

Let $\quad \bar m_p \Big|_{\bar x = \frac{1}{2} - \upsilon \bar p} = \bar m_{h_1}$

and $\quad \bar m_p \Big|_{\bar x = \frac{1}{2} + (1-\upsilon)\bar p} = \bar m_{h_2} = \bar m_p \Big|_{\bar x = \frac{1}{2} - (1-\upsilon)\bar p}$

then

$$\bar m_h = \bar m_{h_1}, \quad \bar m_{h_1} \ge \bar m_{h_2}$$

$$= \bar m_{h_2}, \quad \bar m_{h_1} \le \bar m_{h_2} \tag{11}$$

It can be seen from equation (9) that successive values of $\bar m_p$ at a certain section will be given by the ordinates of the curves

$$\bar M = \bar M(\bar x - s\bar p) \tag{12}$$

at the section. So, lower limit of $\bar m_h$ will be given by the point

Nomenclature

k = change in curvature
k_r = initial residual curvature
k_f = final residual curvature
k_p = highest value of k_f due to pitch effect
k_n = highest value of k_f due to nonhomogeneity effect
l = beam length
x = part of beam length
M = moment of resistance as a function of change in curvature
M_1 = loading moment at mid-span
M_y = yield moment for the section
m_θ = component of loading moment in the plane of initial curvature
m_p = peak value of m_θ
m_h = highest value of m_p
P = force of separation acting on the rolls
p = pitch of helical path followed by a point on the surface of the bar
R = radius of the roll
r = radius of the bar
v_x = throughput speed of the bar
v_t = tangential velocity of any point on the surface of the bar
w = intensity of u.d.l. on the beam length in a line-contact roll
α = angle between cross rolls
β = a nondimensional parameter < 1
ζ = a nondimensional parameter ≤ 1
η = a function of initial residual curvature
φ = a function of change in residual curvature
λ = a dimensionless parameter varying between 0 and 1
θ = angle between the plane of loading and the plane of initial curvature
θ_0 = value of θ for a section at $x=0$
ω = angular velocity of rolls

All variables with a bar over them are nondimensionalized variables, defined in text.

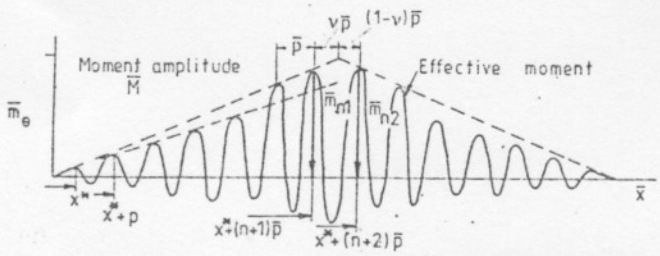


Fig. 4 Oscillating wave for effective moment at a section

of intersection of the curves $\bar{M}(\bar{x})$ and $\bar{M}(\bar{x}-\bar{p})$ (Fig. 5). Let at this point of intersection $\bar{m} = \zeta \bar{M}_1$, then

$$\zeta \bar{M}_1 \leq \bar{m}_h \leq \bar{M}_1, \quad \zeta \leq 1 \qquad (13)$$

It can easily be seen that

$$\zeta = 1 - \bar{p} \quad \text{for point-contact rolls}$$
$$= 1 - \bar{p}^2 \quad \text{for line-contact rolls}$$

Effect of Pitch on Degree of Straightness

It is pertinent to point out here that, so far bending in cross-roll straightening machines is concerned, the bar to be straightened behaves both anisotropically and nonhomogeneously, because the curvature change—moment relationship for the bar changes both with rotation and translation of the bar inside the roll arrangement [6].

Let it be assumed that the loading roll is so adjusted that in a bar with initial curvature $\bar{k}_r^*$ it will produce a change in the residual curvature equal to $\bar{k}_r^*$ in the opposite direction when at the section $\theta = 0$ at $\bar{x} = \frac{1}{2}$ and the corresponding loading moment $\bar{M}_1$ at the loading section $\bar{x} = \frac{1}{2}$ be $1 + \eta^*$.

Then as pointed out earlier [6], even if the bar with initial curvature $\bar{k}_r^*$ is rotated, about its own axis, inside the roll arrangement $\bar{m}_h$ will vary in the range

$$\zeta[1+\eta^*] \leq \bar{m}_h \leq [1+\eta^*] \qquad (14)$$

So, due to variation in $\bar{m}_h$, the final residual curvature will be in the range

$$0 \leq \bar{k}_f \leq \bar{k}_p \qquad (15)$$

where $\bar{k}_p$ is the final residual curvature in case loading takes place from the moment $\zeta[1+\eta^*]$.

If the initial curvature of the bar is in the range $0 \leq \bar{k}_r \leq \bar{k}_r^*$, then the ranges of $\bar{m}_h$ and $\bar{k}_f$ will also be given by (14) and (15) but if a length of the bar with initial curvature $\bar{k}_r' > \bar{k}_r^*$ is passed through the rolls, loading moment $\bar{M}_1$ developed at $\bar{x} = \frac{1}{2}$ will be higher than $(1+\eta^*)$. It can be seen from Fig. 4 that even if there is no pitch effect, there will be a final residual curvature $\bar{k}_f'$ for unloading from the moment $\bar{M}_1'$ (nonhomogeneity effect). Again due to pitch effect if the increase in the final residual curvature is $\bar{k}_f''$, then even if the slope of the curve $\bar{M}(\bar{k}, \bar{k}_r')$ is slightly more than that for the curve $\bar{M}(\bar{k}, \bar{k}_r')$, which is not likely, $\bar{k}_f''$ will not be less than $\bar{k}_p$, i.e.,

$$\bar{k}_f^* = \bar{k}_f' + \bar{k}_f'' > \bar{k}_p$$
$$\text{and } 0 \leq \bar{k}_f \leq \bar{k}_f^* \qquad (16)$$

where $\bar{k}_f^*$ is the value of $\bar{k}_f$ when the bar is unloaded from the moment $\zeta \bar{M}_1$. Hence, it is definitely advisable that unless other difficulties arise the rolls should be adjusted for highest initial curvature expected.

It is quite possible to draw a curve $\bar{M} = \phi(\bar{k}_*)$ (Fig. 7), where $\bar{k}_*$ is the change in residual curvature. Let the inverse relationship be given by

$$\bar{k}_* = \psi(\bar{M}) \qquad (17)$$

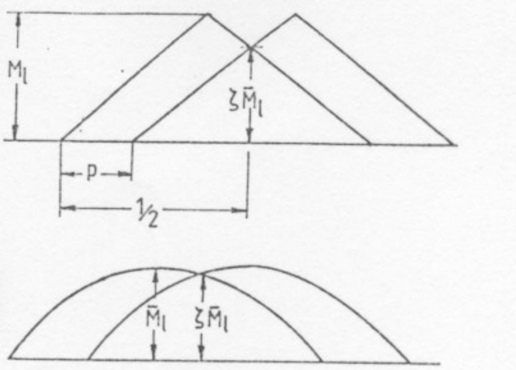


Fig. 5

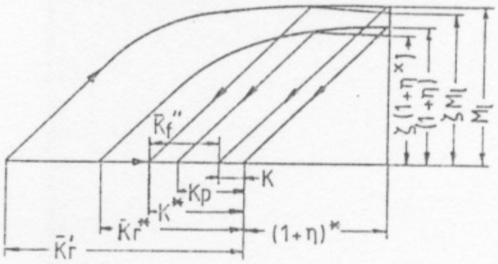


Fig. 6

then from Fig. 8

$$\bar{k}_p = \psi(\bar{M}) \Big|_{\bar{M} = 1+\eta^*} - \psi(\bar{M}) \Big|_{\bar{M} = \zeta[1+\eta^*]}$$

$$\text{or } \bar{k}_p = \bar{k}_* - \psi(\bar{M}) \Big|_{\bar{M} = \zeta[1+\eta^*]} \qquad (18)$$

Since the slope of the curve at $\bar{M} > 1$ is likely to be very high equation (18) shows that slight decrease in ζ, i.e., increase in $\bar{p}$ will cause the value of $\bar{k}_p$ to increase considerably, thus affecting the degree of straightness of the product adversely (Fig. 8).

Evaluation of Final Residual Curvature

Though for a qualitative exploration of the influence of pitch on the degree of straightness of the product equation (18) is of much importance, it may not be convenient to evaluate $\bar{k}_p$ quantitatively with the equation. However, the value of $\bar{k}_p$ may be determined as follows:

(*i*) η^* is calculated for the particular value of $\bar{k}_r^*$ from the equation [see equation (9) of Part I]

$$1 + \eta^* - [\lambda \bar{k} + (1-\lambda)\ \bar{\xi}]_{\bar{k} = 1+\eta^* + \bar{k}_r^*} \qquad (19)$$

(*ii*) $\bar{k} = \bar{k}_1$ is obtained from the solution of the equation

$$\zeta[1+\eta^*] - [\lambda \bar{k} + (1-\lambda)\ \bar{\xi}] = 0 \qquad (20)$$

and finally

(*iii*) $\bar{k}_p$ is calculated from

$$\bar{k}_p = \bar{k}_r^* [\bar{k}_1 - \bar{\xi}\{1+\eta^*\}] \qquad (21)$$

Throughput Speed and Different Straighteners

Equation (3) may be rewritten as

$$v_x = \omega R \left[1 + \left(\frac{2\pi \bar{r}}{\bar{p}}\right)^2\right]^{-\frac{1}{2}} \qquad (3a)$$

From the above equation, it is clear that throughput speed v_x will increase if,

(*i*) ω is increased. But ω cannot be increased beyond a particular limit, because of centrifugal forces developed, due to initial curvature of the bar, at higher angular velocity

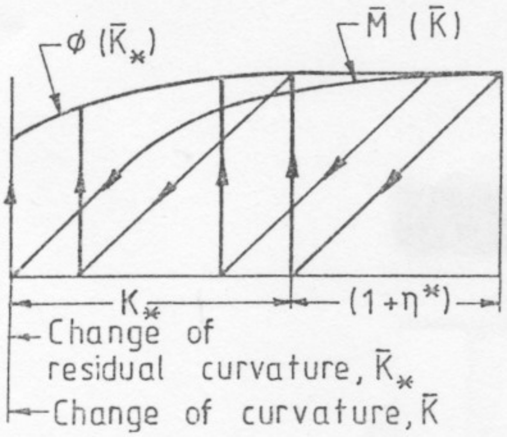


Fig. 7 Moment as a function of residual curvature

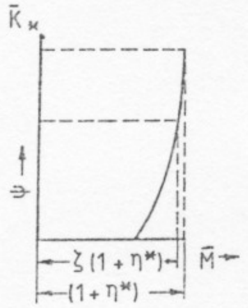


Fig. 8

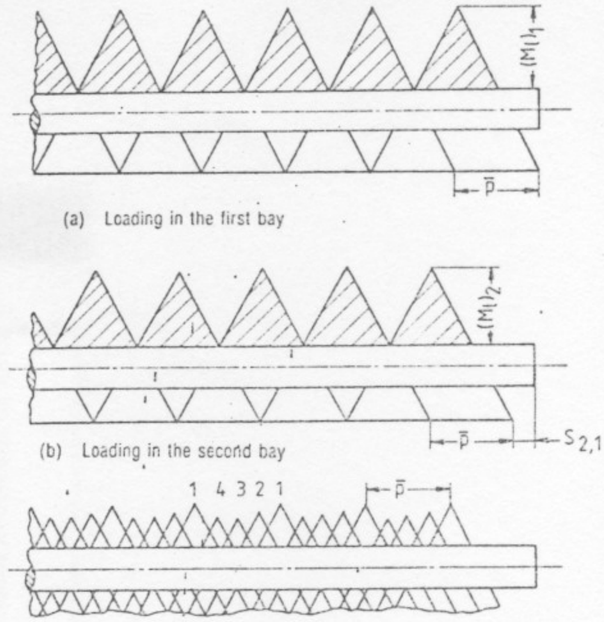
$\dot{\theta} = \omega R \cos \alpha / r$. These centrifugal forces create difficulties in operation, particularly in bars with higher slenderness ratio.

(*ii*) p is increased. To maintain a minimum degree of straightness ζ cannot be lowered beyond a particular value, i.e., $\bar{p}$ cannot be increased beyond a limit. But $\bar{p}$ can be increased, without decreasing ζ, by increasing the beam length l. Precisely for this reason throughput speed in a six-role straightener or cluster-roll straightener, in which the beam length is higher, is higher than that in air-bend straightener. Moreover, due to the distributed nature of loading near the mid-span in case of the six-roll straightener, raises the value of ζ slightly higher than that in the air-bend straightener. But there is a limitation in increasing the beam length also, because there will be a length, slightly less than half the beam length, at each end of the bar, which will not be straightened to the required degree because $\bar{m}_h$ at the sections will not be in the range (14) but instead it will be in the range $0 \leq \bar{m}_h \leq \zeta [1 + \eta^*]$ (Fig. 5).

In case of a line-contact straightener, as mentioned earlier $\zeta = 1 - \bar{p}^2$, so for the same degree of straightness of the product, it has higher $\bar{p}$ than air-bend straighteners, if the pitches in line-contact roll and air-bend straighteners are $\bar{p}_w$ and $\bar{p}_a$, respectively, then

$$\bar{p}_w = (\bar{p})^{\frac{1}{2}} > \bar{p}_a \tag{22}$$

Thus for the same range of $\bar{k}_f$, throughput speed in line contact roll straighteners will be considerably higher than that in air-bend straighteners. But the conditions of contact between the loading roll and the beam length of the bar is uncertain and sometimes loading may be quite away from the assumed loading, leaving higher final residual curvature than it is otherwise expected. However, flatness of the moment amplitude curve near $\bar{x} = \dfrac{1}{2}$, for six-roll straighteners makes ζ slightly higher partially avoiding the bad effect of uncertainty of contact.



(a) Loading in the first bay

(b) Loading in the second bay

(c) Superimposition of loading for all the bays

Fig. 9

However, throughput speed can be increased further without any increase in beam length hence under-straightened beam length at the end avoiding uncertainty in loading, by passing the bar through a number of loading bays, as is done in multistaggered roll straighteners, such that the bar is subjected to loading opposite in sign in adjacent loading bays (phase shifting). If successive loading bays are placed even at arbitrary distances, then a different bay, the corresponding effective moment diagrams will have different phase shifts. If these diagrams for different bays are superposed (Fig. 9), then the top zig-zag curve, enveloping all these diagrams will give the distribution of $\bar{m}_h$ experienced by different sections of the bar during their passage through the loading bays. However, since the final residual curvature in a proceeding bay, the curvature-change moment diagram and M_1 will go on changing as shown in Fig. 10. As a consequence the range of residual curvature will be narrowing down at each bay and the final residual curvature after the last loading bay will indeed be very small. If proper positioning of work rolls is done in a multistaggered roll straightener with n loading bays, then

$$\zeta \simeq \left(1 - \frac{\bar{p}}{n}\right)$$

$$\text{or } \bar{p} \simeq n(1 - \zeta) = n\,\bar{p}_a \tag{23}$$

where $\bar{p}_a$ is the pitch in an air-bend straightener. So, for a given degree of straightness, a pitch which is n times that in air-bend straighteners can be provided to a multistaggered roll straightener thus increasing the throughput speed substantially.

Separating Forces on Rolls

Considering loading in point-contact rolls like air-bend rolls, highest concentrated force P acting on the loading roll is given by

$$P = \frac{4\,M_1}{l} \tag{24}$$

So, if the beam length is smaller, separating force on the rolls is higher. Thus separating force in air-bend or multistaggered roll straightener is higher than that in six-roll or cluster-roll straighteners. Coming to line-contact roll straighteners, if it is

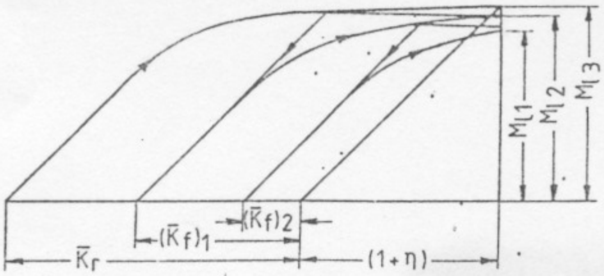


Fig. 10

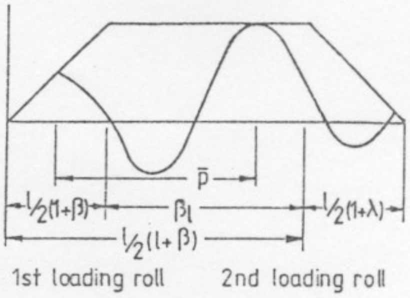


Fig. 11(*a*)

Table 1

Type of Machine	Angular adjustment practically possible
(*1*) Air-bend straightener	5 deg - 10 deg
(*2*) Line-contact roll straightener	10 deg - 20 deg
(*3*) Six-roll straightener	mean value 30 deg
(*4*) Multi-staggered roll straightener	up to 35 deg

to maintain the same degree of straightness of product as air-bend straighteners of same beam length, then

$$\zeta \frac{wl^2}{8} = \zeta \frac{Pl}{4}$$

$$\text{or } wl = 2P \tag{25}$$

where w is the intensity of uniformly distributed loading in line-contact rolls. So, total separating force on the loading roll in line-contact roll straighteners is double of that in air-bend straighteners. So, these machines with short beam length, particularly line-contact roll straighteners must be built with sufficient strength and rigidity to withstand the higher load without adverse effect on the degree of straightness of the product. In case of line-contact roll straighteners, the load is distributed throughout the beam length, achieving a certain amount of rolling mill effect. This rolling mill effect with high transverse compressive stress helps in sizing and ovality correction of the bar, particularly those which are of materials with good flow characteristics and low tensile strength. The full length contact makes these machines suitable for straightening and polishing centerless twined bars, when it is desirable to improve surface finish of the bars, while being straightened.

Discussion and Suggestion

In the above, analysis of the straightening processes in different cross-roll straighteners, which clearly brings out the merits and demerits of different cross-roll straighteners, with reasons, have been made. As regards pitch, the above analysis may be verified with the actual limitations in different machines given in Table 1.

It is also clear from the above that pitch effect plays the most important role in influencing degree of straightness of the product. However, if ζ can be made unity, pitch effect will totally be eliminated. Value of ζ depends on the flatness of the moment-amplitude curve near $\bar{x} = \frac{1}{2}$. So, if the moment amplitude curve is absolutely flat [Fig. 9(*a*)] ζ will be unity, even for a pitch as high as β, thus increasing the throughput speed tremendously, at the same time eliminating the pitch effect. This moment-amplitude curve can be induced by placing two loading rolls one at $\bar{x} = \frac{1}{2}(1-\beta)$ and the other at $\bar{x} = \frac{1}{2}(1+\beta)$ besides the support rolls at $\bar{x} = 0$ and $\bar{x} = 1$ or by

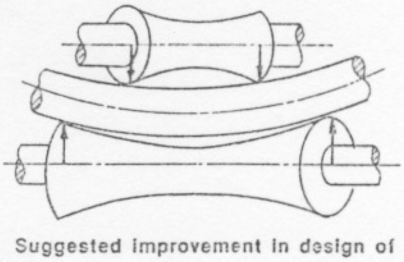


Fig. 11(*b*) Suggested improvement in design of cross-roll straightener

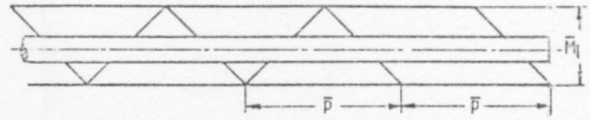


Fig. 11(*c*) Loading moment along the length

an air-bend set, both rolls being concave as shown in Fig. 11(*b*). With such an arrangement of rolls, the highest value of final residual curvature, k_n will be given by

$$\bar{k}_n = 1 + \eta^* - \bar{M}_0(\bar{k})_{\bar{k} = 1 + \eta^*}$$

However, this residual curvature which is small enough can further be reduced by passing the bar through a second loading bay with similar roll arrangement, such that in the succeeding bay

$$\bar{M}_1 = \bar{k}_1 - \bar{k}_n$$

Conclusion

(*1*) An analysis that clearly brings out merits and demerits of different cross-roll straighteners along with their causes has been made.

(*2*) Method of evaluation of standard of straightness has been indicated.

(*3*) The possible improvement in design of roll arrangement for higher throughput speed has been suggested.

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