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NUMERICAL MODELLING OF LIQUID STEEL CONTINUOUS CASTING PROCESSES

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ABSTRACT

The liquid steel fluid dynamics inside the different vessels that form the continuos caster is simulated using a 3D turbulent flow finite element model developed in previous publications. Three different physical mechanisms used to move the liquid steel are analysed: gravity force, electromagnetic forces and the stirring by inert gas injection. The aim of this paper is to show how the numerical models contribute to the understanding, design and optimization of the industrial continuous casting process.

1. INTRODUCTION

The liquid steel flow behavior and the steel solidification determine the quality and productivity of a particular industrial continuous casting installation (see Figure 1).

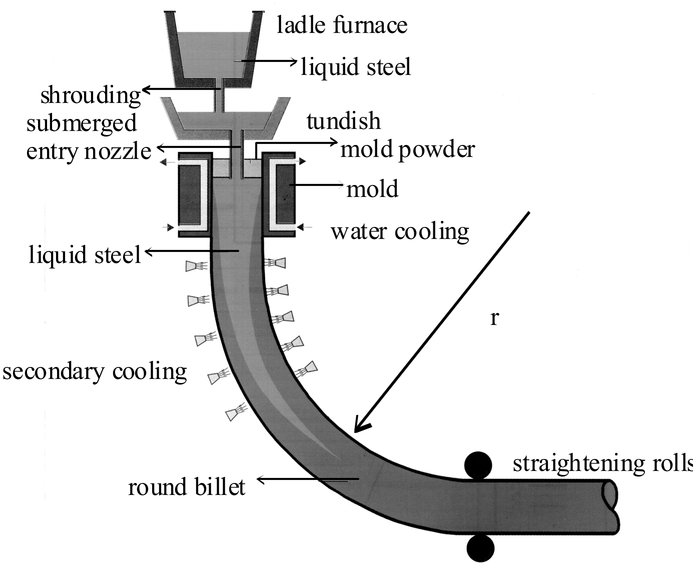

Figure 1 – Typical continuous casting installation

During the manufacturing process liquid steel goes through a set of vessels. These vessels are:

- Ladle: where the addition of alloys takes place.
- Tundish: where the liquid steel is distributed among the different lines, and the inclusions are removed by flotation.
- Nozzles: which connect different vessels (ladle-tundish; tundish-mold).
- Mold: where the steel solidifies taking the final shape.

In these vessels it is important to keep the liquid steel in continuous movement to avoid the cooling and solidification in non-convenient places.
The liquid steel flow is in turbulent regime inside all the continuous casting equipment (see Figure 1). For example, in the submerged entry nozzle typical data are:

- Internal nozzle diameter, $d_b = 0.04$ *to* $0.07\ m$
- Liquid steel flow velocity in the nozzle, $q_b = 0.6$ *to* $1.1\ m/s$
- Liquid steel kinematic viscosity, $\mu/\rho = 9.31E\text{-}7\ m^2/s$. Therefore, the Reynolds number, $Re = (v_b\ d_b\ \rho) / \mu \approx 25\ 800$ *to* $75\ 200$ (well inside the turbulent regime).

The classical (k-ε) turbulence model in conjunction with the method of wall functions developed by Launder and Spalding [1] is usually used to model industrial problems; where k is the turbulent kinetic energy and ε is the turbulent kinetic energy dissipation rate.

To solve the numerical difficulties present in the integration of the partial differential equation system formed by the Navier Stokes, k-transport and ε-transport equations we developed an iterative algorithm, referred to as (k-L)-predictor / (ε)-corrector scheme [2-3]; where L is the length scale. The algorithm was implemented in the finite element code FANTOM [4] and extensively tested to demonstrate its robustness and reliability [2-3,5].

The liquid steel can be moved by three different mechanisms in the steelmaking process:

- ✓ The gravity force: tundish, submerged entry nozzle, mold.
- ✓ The injection of an inert gas: ladle, mold.
- ✓ The electromagnetic forces: ladle, tundish, mold.

In this paper we show a numerical example of each mechanism used to move the liquid steel using our (*k-L*)-predictor / (*e*)-corrector algorithm.

2. TURBULENCE MODEL

Considering viscous incompressible flow, isothermal flow, constant liquid steel properties (density =*r*, laminar viscosity = *m*), buoyancy force (**F***b*), external forces (**F***e*) and the turbulence *k-e* model, the following equations are solved:

$$\tilde{\mathbf{N}}\cdot\mathbf{v}=0 \qquad (1)$$

$$r\frac{\partial \mathbf{v}}{\partial t}+r\mathbf{v}.\tilde{\mathbf{N}}\mathbf{v}-\tilde{\mathbf{N}}\cdot\left[\left(m+m^t\right)\left(\tilde{\mathbf{N}}\mathbf{v}+\tilde{\mathbf{N}}\mathbf{v}^T\right)\right]-\tilde{\mathbf{N}}P+ \\ +r\mathbf{g}+\mathbf{F}_b+\mathbf{F}_e=\mathbf{0} \qquad (2)$$

$$r\frac{\partial k}{\partial t}+r\mathbf{v}\cdot\tilde{\mathbf{N}}k-\tilde{\mathbf{N}}\cdot\left[\left(m+\frac{m^t}{s_k}\right)\tilde{\mathbf{N}}k\right]- \\ -m^t\left(\tilde{\mathbf{N}}\mathbf{v}+\tilde{\mathbf{N}}\mathbf{v}^T\right):\tilde{\mathbf{N}}\mathbf{v}+r\frac{C_m k^2}{m^t/r}-\mathbf{F}^k{}_b=0 \qquad (3)$$

$$m^t=C_m\, r\sqrt{k}\, L \qquad (4)$$

$$r\frac{\partial e}{\partial t}+r\mathbf{v}\cdot\tilde{\mathbf{N}}e-\tilde{\mathbf{N}}\cdot\left[\left(m+\frac{m^t}{s_e}\right)\tilde{\mathbf{N}}e\right]- \\ -r\,C_m C_1\,k\left(\tilde{\mathbf{N}}\mathbf{v}+\tilde{\mathbf{N}}\mathbf{v}^T\right):\tilde{\mathbf{N}}\mathbf{v}+r\frac{C_2 e^2}{k}-\mathbf{F}^e{}_b=0 \qquad (5)$$

$$L=\frac{k^{3/2}}{e} \qquad (6)$$

where **v** is the time averaged velocity; *P* is the time averaged pressure; m^t is the turbulent viscosity; **g** is the gravity force; $\mathbf{F}^k{}_b$ is the correction of *k*-transport equation by the buoyancy forces; $\mathbf{F}^e{}_b$ is the correction of *e*-transport equation by the buoyancy forces; *L* is the mixing length; and the typical constants of the k-ε model of Launder and Spalding [1] are $C_m = 0.09$, $C_1 = 1.44$, $C_2 = 1.92$, $s_k = 1.0$ and $s_e = 1.3$.

To solve these equations we use:

- A standard isoparametric finite element discretization for **v**, *k* and *e*; incompressibility imposed by the penalty method; a Streamline Upwind Petrov Galerkin technique; trapezoidal rule for time dependent problems (Zienkiewicz and Taylor, [6]).
- A *k-L* predictor / *(e)* corrector iterative algorithm (Goldschmit et al, [1-2]).
- Wall functions as boundary conditions (Príncipe and Goldschmit, [7]).
- The corrections of *k* and *e* transport equation by the buoyancy forces (Goldschmit and Coppola Owen, [8]).

3. MOVEMENT BY INJECTION OF AN INERT GAS

The stirring of liquid steel contained in a ladle is an example of a flow driven by inert gas injection. The ladles are near-cylindrical vessels (radius, *R » 2 - 3 m*; height, *H » 2 - 4 m*) that contain liquid steel and a surface slag layer (*0.1 – 0.4 m*) to avoid the reoxidation of steel. In order to obtain a quick chemical and thermal homogenisation it is essential to predict where it is convenient to place the gas injection nozzle and to determine the ideal gas flow rate (Q_g).

A general scheme of liquid steel movement caused by injection of inert gas in the ladle is shown in Figure 2.

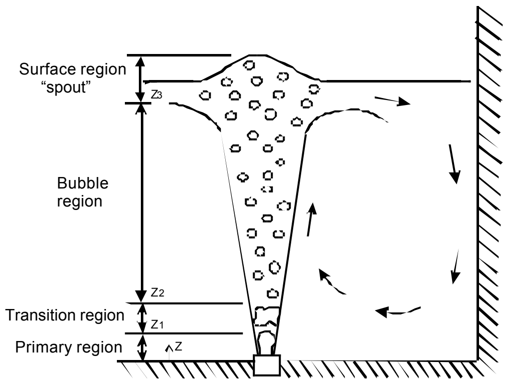

Figure 2 – Description of the fluid dynamics in the gas stirred ladle

Gas is injected into the liquid steel through a porous nozzle where bubbles are formed; the bubbles rise in the liquid and break up into smaller bubbles and determine two zones:

<u>Two-phase plume zone</u> :

- Primary or momentum region where the flow is governed by inertia forces.
- Transition region where the gas loses its kinetic energy and disintegrates in small bubbles.
- Bubble region where the bubbles rise by the effect of density difference between the gas and liquid steel.
- Surface region, it is the zone closest to the surface.

The primary and transition region in the plume occupy a very small volume of the industrial ladle (*Z_2 » 1 to 3 mm*; [9]). Therefore, the buoyancy forces control the fluid dynamics in this zone.

The plume zone is treated as a *pseudo-one phase* [10-12] with a lower density,

$$r=a\,r_g+\left(1-a\right)r_l \qquad (7)$$

where *a* is the gas fraction, r_g is the gas density and r_l is the liquid density. The gas fraction is estimated according to the drift flux model [12] as,

$$\alpha = \frac{Q_1 - \pi r_{pl}^{\,2}\, \alpha (1-\alpha) U_s}{2\pi \int_0^{r_{pl}} v_{pl}\, r\, dr} \tag{8}$$

$$r_{pl} = 1.5 b \quad ; \quad v_{pl} = 4.5\, Q_g^{\,0.333} H^{0.25} R^{-0.25} \tag{9}$$

$$U_s = 1.08 \left(\frac{g\, d_b}{2} \right)^{0.5} \qquad \text{for steel}$$

$$d_b = 0.569 \left(Q_1\, d_o^{\,0.5} \right)^{0.289} \quad ; \quad b = 0.28\, (z + H_0)^{7/12} \left(\frac{Q_1^{\,2}}{g} \right)^{1/12} \tag{10}$$

$$H_0 = 4.5 \sqrt{d_o} \left(\frac{Q_0^{\,2}}{g} \right)^{0.1} \tag{11}$$

where $Q_1 = Q_g \dfrac{T_l}{T_g} \dfrac{(P_{atm}/g\rho_l)}{[(P_{atm}/g\rho_l) + H - z]}$; T_l and T_g are the liquid and gas temperatures [°K]; P_{atm} is the atmospheric pressure [N/m^2]; d_o is the injection orifice diameter [m]; z is the height co-ordinate of the point; and, $Q_0 = Q_1(z=0)$.

<u>Recirculation zone</u>: it is formed outside the plume zone. The turbulent convective flow controls the movement in this zone. The recirculation zone is treated as liquid steel ($\rho = \rho_l$); in the steady state Navier Stokes equation (2), $\rho\,\mathbf{g} = \mathbf{0}$; $\mathbf{F}_e = \mathbf{0}$ and the buoyancy forces are

$$\mathbf{F}_b = \rho\, \mathbf{g}\, \alpha \tag{12}$$

The corrections of the steady state k-transport (3) and ε-transport (5) equations $\mathbf{F}^k{}_b$ and $\mathbf{F}^\varepsilon{}_b$ introduced in our reference Goldschmit and Coppola Owen [8] to model the buoyancy effect produced by a change of gas fraction in the direction of gravity are

$$\mathbf{F}^k{}_b = \mu^t\, \mathbf{g} \cdot \nabla \alpha \tag{13}$$

$$\mathbf{F}^\varepsilon{}_b = \rho\, C_\mu\, C_1\, k\, \mathbf{g} \cdot \nabla \alpha \tag{14}$$

The validation of our quasi single phase (k-L)-predictor / (ε)-corrector turbulent model was performed using different water model experiments published in the bibliography and reported in our previous publications [8,13].

<u>Industrial application</u>

Nowadays most industrial ladles use two porous plugs but there is no clear preference regarding their position in the ladle bottom. The numerical model can therefore be used to understand the influence of the radial position of the plugs and the angle between them on the fluid flow inside the ladle. In this work the effect of the angle between the porous plugs (see Figure 3) for a radial position *r/R = 0.72* in a 200 ton industrial ladle is analysed. Taking into account symmetry conditions only half, or even one quarter, of the ladle is modelled.

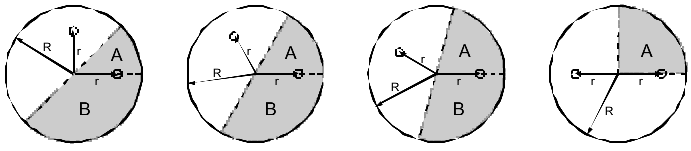

Figure 3 – Different angular positions of the plugs (90°, 120°, 150° and 180°)

The modelled ladle has a bottom diameter of *3.3m* , a top diameter of *3.5m* and the liquid steel height is *3.5m.* Three gas flow rates (Q_g) are analysed: *50*, *500* and *1000* lt/min per plug. Taking into account symmetry conditions only half of the ladle is modeled. For the ladle with a *180°* separation between the plugs a second symmetry plane exists and therefore only one quarter of the ladle is modeled. The biggest mesh used is composed by 7904 linear hexahedral elements and the smallest one by 3648. The computational times using a Sun Ultra Enterprise 450 workstation and a convergence criteria of *0.1%* for the mixing length L2 norm are 24 and 4 hours respectively.

In order to describe the effect of the angle between the plugs on the resulting liquid steel flow, the modelled zone can be divided in two regions, A and B (see Figures 3 and 4). As the angle between the plugs increases the size of region A increases while that of region B is reduced.

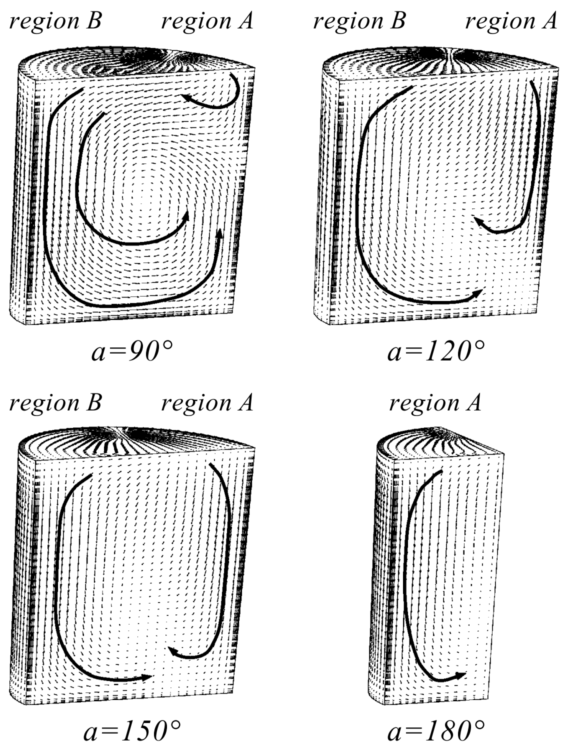

Figure 4 – Velocity distribution

For an angular separation of 90° between the plugs the size of region B is much greater than that of region A and therefore all of the steel that has been lifted by the gas bubbles descends in region B. When the angle is increased to

120° part of the steel starts to flow downwards in region A but the recirculation loop formed in this region is weaker than the one found in region B and it does not reach the bottom of the ladle. For 150° the size of both regions is similar and both recirculation loops reach the ladle bottom. For 180° regions A and B are symmetric and therefore the flow pattern is the same in both of them.

In order to have a parameter that allows a comparison between the different configurations, the recirculation flow rate is defined as the total amount of liquid that flows upwards or downwards through the horizontal mid plane. In Figure 5 the variation of the recirculation flow rate with the Ar flow rate per injector is shown. It can be seen that the recirculation flow rate for $\alpha = 90°$ is much greater than for the rest of configurations.

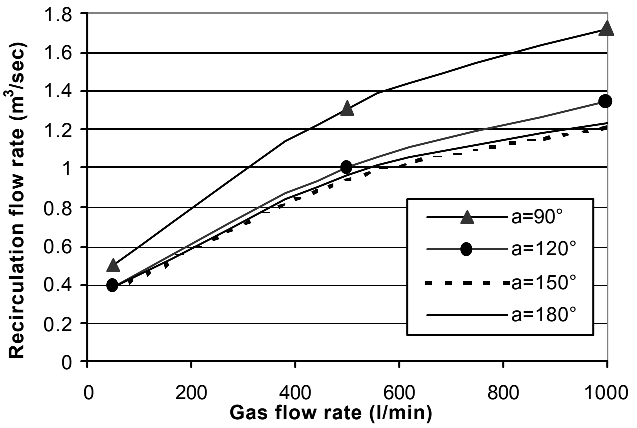

Figure 5 – Recirculation flow rate

Another parameter that can be used to compare the different ladles is the percentage of dead volume defined as the percentage of the total volume of the ladle that has velocities below a certain critical velocity (see table 1).

Q_g [lt/min]	50	500	1000
Critical velocity [m/sec]	0.04	0.10	0.14

Table 1 – Critical velocities

In accordance with the results for the recirculation flow rate, the lowest percentage of dead volume is obtained in the ladle with an angle of 90° between the plugs (see Figure 6).

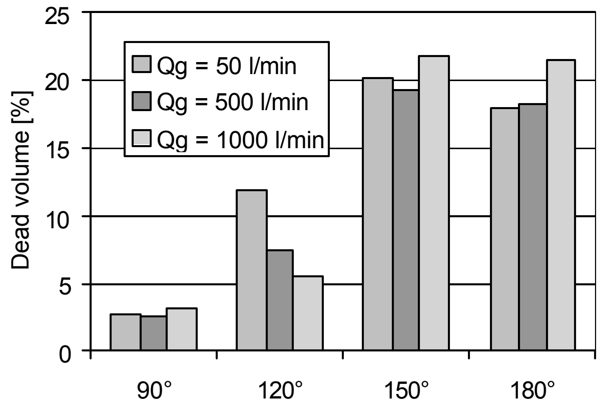

Figure 6 – Dead volume

The high recirculation flow rate and low percentage of dead volume observed for $\alpha = 90°$ can be attributed to the fact that, as only one recirculation loop is formed, the liquid steel is allowed to move more freely and the viscous forces are lower.

Taking into account that both an increase in recirculation flow rate and a decrease in the percentage of dead volume favour mixing inside the industrial ladle, the numerical results show that a *90°* angle between the plugs can improve the efficiency of the inert gas stirring process.

4. MOVEMENT BY ELECTROMAGNETIC FORCES

In this section we describe the stirring of the liquid steel that flows in the continuous casting round mold by the application of an external electromagnetic force. Such a force is obtained by a time dependent electromagnetic field produced by a set of coils connected in pairs. The electromagnetic force can be evaluated as

$$\mathbf{F}_e = \mathbf{J} \times \mathbf{B} \qquad (15)$$

where J is the current density and B is the magnetic induction. The current density depends on the steel movement through the constitutive relation

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad (16)$$

where v is the steel velocity and σ is the electric conductivity. In the second term of the above equation we can see a coupling with the steel motion. However this coupling can be neglected when the magnetic Reynolds number, defined as

$$Re_m = v\, L\, \mu \sigma \qquad (17)$$

where L a characteristic length of the problem, is much smaller than 1 [14]. We present results for bar diameter of *0.215* and *0.290 m*. In this case the Reynolds magnetic number is *0.03* and *0.07*, then the above approximation can be done. The electromagnetic force has been calculated by the CINI' s Physics Department using a 3D finite element model [15-16]. Using this result we solve the k-ε model equations (1) to (6) in steady state conditions with F_e as external force and $F_b = F_b^k = F_b^\varepsilon = g = 0$.

<u>*Industrial application*</u>:

In Figure 7 we present the distribution of the velocity modulus inside the round mold for an applied current of 6 Hz frequency and 70 A intensity through the coils.

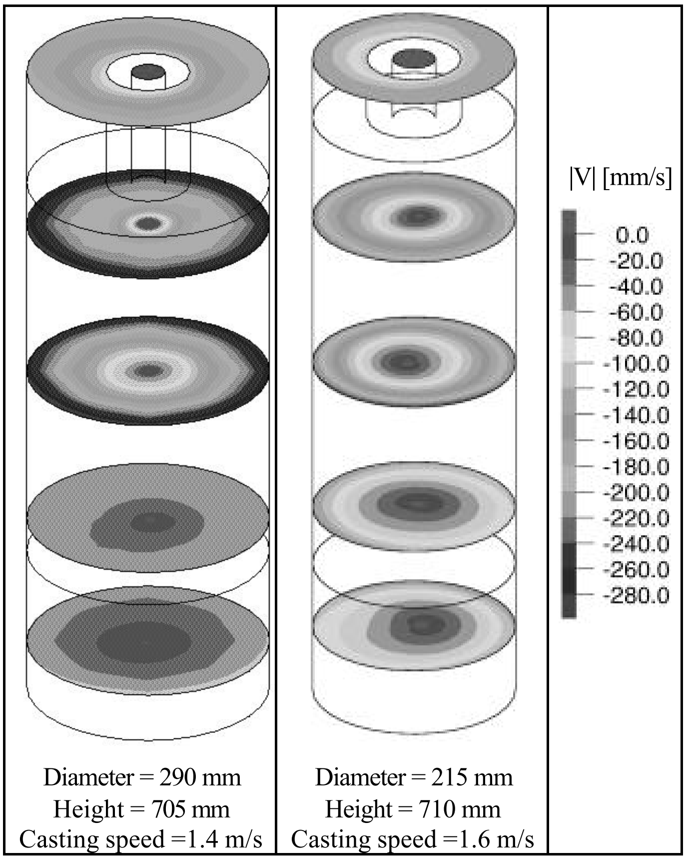

Figure 7 - Velocity modulus

Using this model we are able to predict the flow pattern as a function of the current intensity and frequency.

It is important to avoid high velocities in the meniscus zone (the steel free surface zone supposed to be plane) because powder entrapments at the layer covering the steel surface and lubrication defects may occur. In Figure 8 the meniscus velocity as function of the current intensity, for a 6Hz frequency current is shown

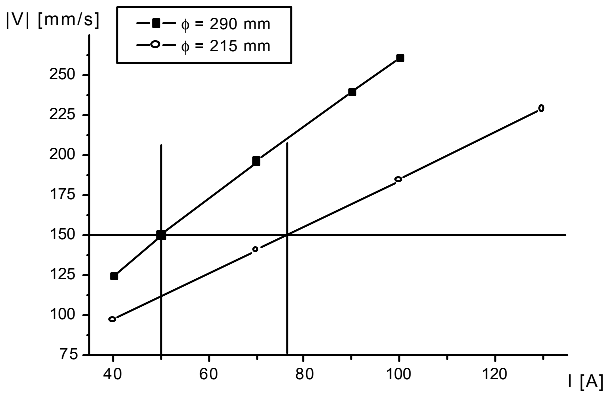

Figure 8 - Meniscus velocity as a function of current intensity

If a 50 A current intensity applied to a 290 mm bar produces a good result, then we can say that 150 mm/s is a good velocity modulus at the meniscus and 75 A current intensity is convenient for a 215 mm bar.

Determination of the optimum current intensity for a given bar diameter is a time and money consuming process based on trial and error. Therefore, the possibility of counting with a numerical tool that can guide the current intensity selection process has been welcome by the industrial plants.

The results have been used successfully by several industrial facilities of the Tenaris Group (Siderca, Argentina; Dalmine, Italy; Tamsa, Mexico).

5. MOVEMENT BY GRAVITATIONAL FORCE

The quality of the steel and the productivity of the caster depend strongly on the characteristics of the flow inside the different vessels of the caster. The movement of liquid steel along the continuous caster is mainly driven by gravity. Besides, gravity determines the profile and dynamics of the steel surface, which is of particular importance in the performance of the caster.

When solving flow problems with free surface (or an interface between two fluids), the location of the interface is also an unknown to be determined as part of the solution. In this paper the pseudoconcentration method was used to track the position of the interface [17].

This method solves flow equations without buoyancy and external forces ($\mathbf{F}_b = \mathbf{F}_e = \mathbf{F}^k{}_b = \mathbf{F}^e{}_b = \mathbf{0}$) inside an extended domain. The interface of the fluid is supposed to stay inside this fixed domain along the whole numerical simulation. To track the interface, a new variable, C, is introduced. This variable (called pseudoconcentration) satisfies the following advection equation,

$$\frac{\partial C}{\partial t} + \mathbf{v} \cdot \tilde{\mathbf{N}} C = 0 \tag{18}$$

which is solved using isoparametric finite elements with Streamline Upwind Petrov Galerkin technique [6].

An arbitrary value of the pseudoconcentration C_c is defined as follows:

- $C(\mathbf{x},t) > C_c$ if $\mathbf{x}$ is in the region occupied by fluid 1
- $C(\mathbf{x},t) < C_c$ if $\mathbf{x}$ is in the region occupied by the fluid 2
- $C(\mathbf{x},t) = C_c$ if $\mathbf{x}$ correspond to the interface between fluid 1 and fluid 2

The initial value of C *is* set according to the initial position of the interface. Then, the value of C for each time is determined by equation (18) and the position of the interface is given by those $\mathbf{x}$ such that $C(\mathbf{x},t) = C_c$.

In problems with an interface between a fluid and air - free surface problems - the same procedure can be used. In these cases density and viscosity are chosen in such a way that the effect of the air (a pseudofluid) on the real fluid are negligible.

In order to prevent numerical errors due to the presence of strong gradients of pseudoconcentration inside the domain, the pseudoconcentration $C(\mathbf{x},t)$ must be a smooth function of position. A smooth initial condition is imposed on the whole domain. To keep the smoothness of $C(\mathbf{x},t)$, the distribution of pseudoconcentration is re-calculated in each time step (after updating the position of the free surface) using the expression, $C(\mathbf{x},t)=C_c + d \quad s \; sgn(C(\mathbf{x},t)-C_c)$; where d is the distance to the free surface and s is an arbitrary constant ($s = 0.015$ in our simulations).

The validation of our turbulence algorithm in problems with interfaces was performed using different water model experiments published in the bibliography and reported in previous publications [18].

Industrial application

The industrial problem to be considered is the drainage of a ladle. In this case, the most important point is to analyse the surface deformation process. For this problem the whole time evolution of the process is studied and the mixing length model is used to simulate turbulent effects.

In the mixing length model, turbulent viscosity is obtained from the velocity gradient through the expression

$$m^t = r l^2(\nabla \mathbf{v} : \nabla \mathbf{v} + \nabla \mathbf{v} : \nabla \mathbf{v}^T)^{1/2} \qquad (19)$$

where l is the mixing length which must be set a priori. Equation (19) replaces equations (3-6) of the k-ε formulation. Since no differential equations are required to estimate the turbulent viscosity, the mixing length model is called a "zero equation" model.

The steel in the ladle is covered with a layer of slag to prevent its contact with air. At a certain point during the ladle teeming process, part of the slag is carried to the tundish and the draining must be stopped. A large amount of steel may still remain in the ladle. The slag carryover takes place when the steel surface deflects in the form of a funnel, which enters the discharge of the ladle. This always occurs when the ladle in the final stages of the teeming process, with a short column of steel left unteemed. However if initial rotational velocities are high enough, vortex formation may cause slag carryover when there is still a high column of steel in the ladle.

The industrial ladle to be considered has its discharge close to the wall, for it is a well-known fact that eccentricity prevents or delays vortex formation. Also the floor of the ladle is inclined towards the discharge, to reduce the amount of steel left in the ladle when the critical height is reached (the critical height is the height of the unperturbed interface when the slag carryover takes place).

Results are presented in Figure 9. A mesh composed by 3540 hexahedral elements is used. The last two minutes of the drainage process are simulated using implicit time discretization with a time step of 0.01 seconds.

No vortex is formed and the slag carryover occurs when the height of the steel column is comparable to the diameter of the discharge, which is in agreement with observations reported in literature [19-20]

The height of the steel column left in the ladle according to this simulation is about 8 cm over the nozzle. Considering the ladle geometry and dimensions this height corresponds to approximately 2 tons of wasted steel, which compares favorably with usual values reported by plant.

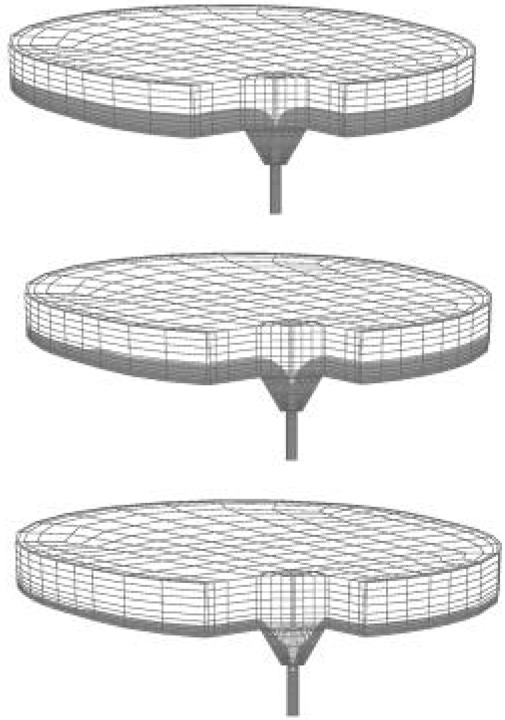

Figure 9 - Teeming of an industrial ladle.

Water model

In order to analyse vortex formation a water model example was analysed. A cylindrical vessel with centred discharge and initial rotational velocity was considered. Results are shown in Figure 10. This mesh consisted of 10600 hexahedral elements.

An implicit scheme was used for time discretization. 45 seconds of the teeming process were simulated with a 0.01 seconds time step.

The figure shows that the irruption of the deformed free surface into the ladle occurs when there is still a high column of water in the ladle. This is in agreement with results observed in experimental models and reported in the literature [21-22].

New experiences are presently being carried out at Buenos Aires University for a quantitative comparison of results.

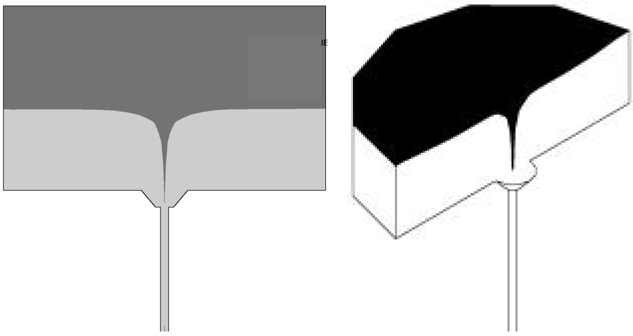

Figure 10 – Simulation of vortex formation in a water model of a cylindrical ladle with initial rotation.

6. CONCLUSIONS

A previously developed Finite Element model for 3D turbulent flows [2,3,5,8] has been applied to different steps in the continuos casting of steel, proving to be a worthy tool for understanding and improving the processes. This is of special importance in a process with liquid steel where visual opacity and high temperatures make measurements difficult and expensive.

For the first vessel in the process, the ladle, the numerical results have cleared the influence of the position of the porous plugs on the resulting liquid steel flow during inert gas stirring. The information can be used to design a new ladle with improved mixing.

The simulations of the flow inside the round mold when the liquid steel is stirred by an external electromagnetic force proved to be a helpful tool for determining the optimum operating conditions. Without the numerical results, the current intensity for each bar diameter had to be selected by a time and money consuming trial and error method. A better understanding of the fluid dynamics inside the mold allowed extrapolating the results from one bar diameter to another one in a much shorter time.

Finally, the ladle teeming simulation showed that, under normal industrial conditions, vortex formation is not expected. The slag carry over takes place due to appearance of a drain sink when the steel column above the nozzle is approximately equal to the diameter of the nozzle.

Faced upon these simulations, numerical methods turn out to be a useful tool in the analysis, design and optimization of liquid steel processes.

ACKNOWLEDGEMENTS

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