

APPLICATIONS OF A (k-ε) MODEL FOR THE ANALYSIS OF CONTINUOUS CASTING PROCESSES

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ABSTRACT

The finite element solution of the turbulent Navier - Stokes equations developed via (k-ε) turbulence models was addressed in previous publications¹⁻⁴, where a (k-L)-predictor / (ε)-corrector iterative algorithm was developed. It was shown that the developed algorithm is robust and converges for the analyses of different flows without requiring the implementation of ad hoc numerical procedures. Using the velocity distributions determined from the solution of the turbulent Navier - Stokes equations the turbulent convection-diffusion transport equations are solved. The dispersion of a die in a turbulent flow can therefore be modeled and the obtained dispersion patterns are validated via flow visualizations in water models. In the present paper, the developed analysis capability is applied to the analysis of continuous casting processes.

KEY WORDS: Turbulence Models, Continuous Casting, Computational Fluid Dynamics

1. INTRODUCTION

The liquid steel flow behavior largely determines the quality and productivity of a steelmaking process. The analysis of liquid steel flows in continuous casting installations using water models and numerical models⁵⁻¹³ increase the knowledge on a particular industrial installation, enabling engineers to:

- study the effect of different variables on the process output;
- increase the process efficiency as regards materials and energy consumptions;
- improve the equipment design;
- etc.

To depict the fact that most of the liquid steel flows, in the facility in Figure 1, are in the turbulent regime we consider the submerged entry nozzle (SEN) example:

- internal nozzle diameter, $d_b = 0.04 \text{ m}$;
- liquid steel flow rate in the mold, $q_b = 0.001 \text{ m}^3/\text{s}$;
- mean velocity in the nozzle, $v_b = q_b / (\pi d_b^2 / 4) = 0.796 \text{ m/s}$;
- kinematic viscosity, $\mu/\rho = 9.31E-7 \text{ m}^2/\text{s}$;
- Reynolds number, $Re = (v_b d_b \rho) / \mu = 34200$ (well inside the turbulent regime)

The main characteristics of the turbulent flow is the random fluctuations in the variables (fluid velocity, pressure, etc.) so that statistically distinct averages can be discerned. The fluctuations are due to a continuous generation and movement of eddies in the flow. It is worth pointing out that a turbulent flow is described by the usual continuum mechanics equations because the smallest length scales in a turbulent

flow are much higher than the molecular length scales.

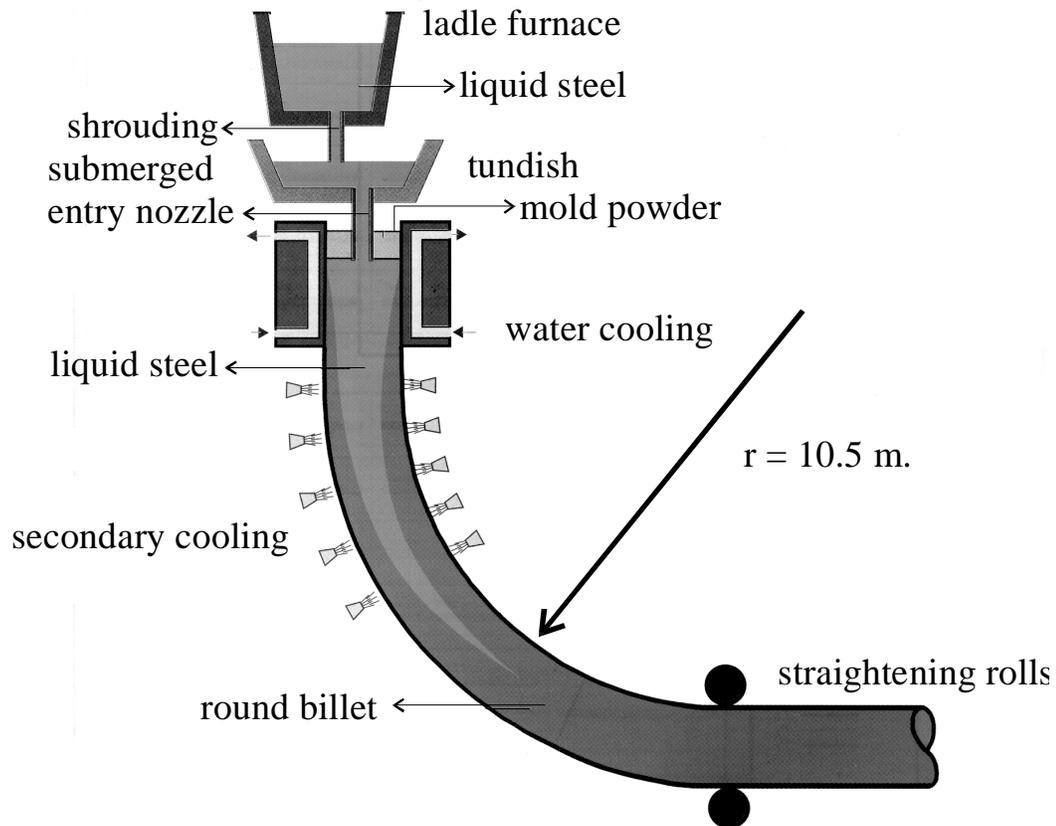


Figure 1.- Round bars continuous casting installation

The mathematical description of a turbulent flow using the time averaged Navier-Stokes equations leads to the development of turbulence models to close the problem. These turbulence models do not simulate the details of the turbulent motion but only the effect of turbulence on the mean-flow behavior.

The classical (k - ϵ) turbulent model in conjunction with the method of wall functions developed by Launder and Spalding¹⁴ is usually used to model liquid steel flows in continuous casting installations⁵⁻¹³; where k is the turbulent kinetic energy and ϵ is the turbulent kinetic energy dissipation rate.

The literature¹⁵⁻¹⁸ discusses the important numerical difficulties present in the integration of the partial differential equation system formed by the Navier Stokes, k -transport and ϵ -transport equations. To solve these numerical difficulties the first Author developed an iterative algorithm, referred to as (k - L)-predictor / (ϵ)-corrector scheme¹⁻⁴; where L is the length scale. The algorithm was implemented in the finite element code FANTOM¹⁹ and extensively tested to demonstrate its robustness and reliability.

In this iterative scheme a predictor result is obtained using a k - L model and it is afterwards corrected integrating the ϵ -transport equation. The iterative scheme loops between predictor and corrector phases until convergence is achieved in the length scale.

When using water models to simulate different continuous casting processes²⁰⁻²², the

flow is visualized by injecting a dye or another substance upstream and then using photographs, conductivity measurements, etc. to study the transport of the injected substance by the fluid flow. To match the experimental determinations when using mathematical models a convection-diffusion equation^{23,24} is solved to describe the transport of the injected substance by the incompressible turbulent flow.

In this paper three industrial applications will be considered:

1. The analysis of different options for the SEN in the mold of a round bars continuous casting mill at SIDERCA (Campana, Argentina) and TAMSA (Veracruz, Mexico).
2. The analysis of one SEN design operating with different mold sizes in the slabs continuous casting at SIDERAR (San Nicolas, Argentina).
3. The tundish of a continuous casting installation at SIDERCA.

2. (k-L)-PREDICTOR / (ε)-CORRECTOR ITERATIVE SCHEME

The proposed iterative scheme solves the k - ϵ model looping between a predictor phase and a corrector phase.

2.1. Hypotheses of the model

The hypotheses of steady state, viscous incompressible flow, constant physical properties, isothermal flow and absence of external loads are used to model k - ϵ turbulent flow (Launder and Spalding¹⁴).

2.2. (k-L) Predictor phase

The predictor phase is the solution of a (k - L) turbulence model.

Continuity equation

$$\underline{\nabla} \cdot \underline{v} = 0 \quad (1)$$

Navier Stokes equation

$$\rho \underline{v} \cdot \underline{\nabla} \underline{v} - \underline{\nabla} \cdot \left[\left(\mu + \mu^t \right) \left(\underline{\nabla} \underline{v} + \underline{\nabla} \underline{v}^T \right) \right] + \underline{\nabla} P = \underline{0} \quad (2)$$

Transport equation for the turbulent kinetic equation (k)

$$\rho \underline{v} \cdot \underline{\nabla} k - \underline{\nabla} \cdot \left[\left(\mu + \frac{\mu^t}{\sigma_k} \right) \underline{\nabla} k \right] - \mu^t \left(\underline{\nabla} \underline{v} + \underline{\nabla} \underline{v}^T \right) : \underline{\nabla} \underline{v} + \rho \frac{C_\mu k^2}{\mu^t / \rho} = 0 \quad (3)$$

Turbulent or eddy viscosity (μ^t)

$$\mu^t = C_\mu \rho \sqrt{k} L \quad (4)$$

The length scale, L

$$L = \frac{k^{3/2}}{\epsilon} \quad (5)$$

where

ρ : density

μ : molecular viscosity

\bar{v} : time averaged velocity

$P = p + 2/3 k$; p is the time averaged flow pressure

The empirical constants of the (k - L) model are the typical constant determined by Launder and Spalding¹⁴: $C_\mu = 0.99$; $\sigma_k = 1.0$.

To solve this predictor phase the ε value from the previous corrector phase is used.

2.2.1. Wall functions

The viscous affected near-wall region where the k - ε model is no longer valid is handled using the well-established “wall function method” of Launder and Spalding¹⁴ which has been extensively used with his turbulence model²⁵⁻³¹.

The first nodes of the finite element mesh close to the walls are placed at a distance Δ from the walls and the velocity component parallel to the wall at the node P is given by

$$\frac{U_P}{U_*} = \frac{1}{\kappa} \ln \left(\frac{EU_*\Delta}{\mu/\rho} \right) \quad (6)$$

where

U_P : parallel velocity to the wall at the node P

U_* : friction velocity

E : function of the wall surface roughness (for smooth surfaces, $E = 9$)

κ : von Karman constant

The boundary layer, $0 < y < \Delta$ (where y is the distance of the wall), is in local equilibrium under a constant shear stress (τ_ω),

$$\tau_\omega = \rho U_*^2 \quad (7)$$

The following relations are used at the node P (Δ of the wall),

$$k_P = \frac{U_*^2}{C_\mu} \quad (8)$$

$$\varepsilon_P = \frac{U_*^3}{\kappa\Delta} \quad (9)$$

In the iterative predictor phase, with U_P obtained from the previous iteration in the k - L model, U_* is calculated from equation (6) using the Newton-Raphson technique for each node P near to the wall. The values of τ_ω and k_P are calculated using the equations (7)-(8) and used as boundary conditions for the following predictor iteration of the k - L model.

The velocity component normal to the node P near to the wall is set to zero^{1-4,25,27,28,30,31} ignoring the mass transfer in the viscous sublayer. An alternative approach reported in the literature²⁸ is to approximate this mass transfer by evaluating the normal component of the velocity from continuity considerations³².

The limitation of the approach adopted in this paper becomes more apparent where the complexity of the flow increases; in particular these limitations are very severe when there is a subtle separation, as shown by Haroutinian and Engelman³³ who compared the wall function method with an alternative approach developed by them

which seems to extend the applicability of the k-ε model. The limitation is shown using the (k-L)-predictor / (ε)-corrector scheme in the plane U-channel example where the location of the separation point in the recirculating zone is not geometrically determined².

2.3. (ε) Corrector phase

In this phase we solve the transport equation for ε,

Transport equation for the turbulent kinetic energy dissipation rate (ε)

$$\rho \underline{v} \cdot \nabla \varepsilon - \nabla \cdot \left[\left(\mu + \frac{\mu^t}{\sigma_\varepsilon} \right) \nabla \varepsilon \right] - \rho C_\mu C_1 k (\nabla \underline{v} + \nabla \underline{v}^T) : \nabla \underline{v} + \rho \frac{C_2 \varepsilon^2}{k} = 0 \quad (10)$$

The empirical constants¹⁴ are, $C_1 = 1.44$; $C_2 = 1.92$; $\sigma_\varepsilon = 1.3$.

The boundary conditions at the nodes P associated to wall functions are ε_p , equation (9).

2.4. Iterative algorithm

The proposed iterative scheme loops between a predictor phase (k-L model) and a corrector phase (ε transport equation) and updates the value of L using an underrelaxation technique (see Table 1).

<u>Starting procedure</u>	$i = 0$ (i is iteration counter) $\underline{v}^{(0)}$, $k^{(0)}$ and $\varepsilon^{(0)}$ (initial guess, for example from a zero equation model solution).
(a) $i = i + 1$	
<u>(k-L)- Predictor phase</u>	$j = 0$ (j is predictor iteration counter)
(b)	$j = j + 1$ Solve Navier Stokes equations (eq. (1)-(2); using a penalty formulation to impose the incompressibility constraint ³⁴). Solve the wall function (eq. (6) for each node associated to a wall, using the Newton Raphson technique determined U^*). Determine τ_w , k_p and ε_p from equations (7)-(9). Update μ^t from equation (4). Solve the transport equation of k (eq. (3)). Update μ^t from equation (4). Repeat from (b) until convergence of k-L model is achieved
<u>(ε)- Corrector phase</u>	$m = 0$ (m is corrector iteration counter)
(c)	$m = m + 1$ Solve the transport equation of ε (eq. (10)). Repeat from (c) until convergence of ε is achieved
	$L^{(i)} = (\alpha - 1) L^{(i-1)} + \alpha (k^{(i)})^{3/2} / \varepsilon^{(i)}$; $0 < \alpha \leq 1$
	Update μ^t from equation (4).
	Repeat from (a) until convergence is achieved in an L -norm.

TABLE 1 - Iterative algorithm

The transport equations are weighted using the Streamline Upwind Petrov Galerkin

technique³⁵⁻³⁷ and a standard isoparametric finite element discretization for \underline{v} , k and ϵ ³⁴.

The underrelaxation used to update L causes a smoother behaviour of the turbulent viscosity, calculated as $\mu' = C_{\mu}\rho\sqrt{k}L$, than the behaviour of the turbulent viscosity, calculated as $\mu' = \frac{C_{\mu}\rho k^2}{\epsilon}$, obtained when using a classical k - ϵ model.

When we fix the counter j and m in zero and $\alpha = 1$, our algorithm is transformed in a classical staggered solution of the k - ϵ model.

3. TRANSPORT OF A SUBSTANCE

The analysis of a tracer dispersion was carried out by solving an additional time-dependent concentration equation:

$$\frac{\partial c}{\partial t} + \underline{v} \cdot \nabla c = \nabla \cdot [(D + D') \nabla c] \quad (11)$$

where

c : the time averaged concentration of the injected substance

D : the molecular diffusivity

D' : the turbulent diffusivity; $D' = \frac{\mu'}{\rho}$

The equation (11) is a typical convection-diffusion equation in transient state with a non constant diffusion coefficient. It is integrated with the Streamline Upwind Petrov Galerkin technique³⁵⁻³⁶ and a standard 8-node (3D) isoparametric finite element discretization³⁴ for c .

4. CONTINUOUS CASTING APPLICATIONS

In this Section we apply the CFD tools presented above to the analysis of three problems related to industrial continuous casting installations for steel products.

4.1. Submerged entry nozzles

The output of the numerical models is used to judge the SEN designs taking into consideration the following requirements:

- the steel flow entering the mold should be routed to the meniscus in order to increase its temperature³⁸.
- The flow at the mold exit should be as close as possible to a plug flow³⁸.
- The flow should not disturb the surface in order not to remove the casting powders³⁸.
- The flow should transport non metallic inclusions to the casting powders³⁹.
- To avoid clogging the flow inside the SEN should not present recirculation zones⁴⁰.

4.1.1. Design of a SEN for a round mold

The liquid steel flow inside a round mold was analyzed for five SEN designs:

- *Design A* : direct out-flow (1 hole)
- *Design B* : lateral out-flow, straight holes (4 holes)
- *Design C* : lateral out-flow, inclined holes (4 holes)
- *Design D* : combination of A + B (5 holes)

In figure 2 we show the scheme nozzle/mold.

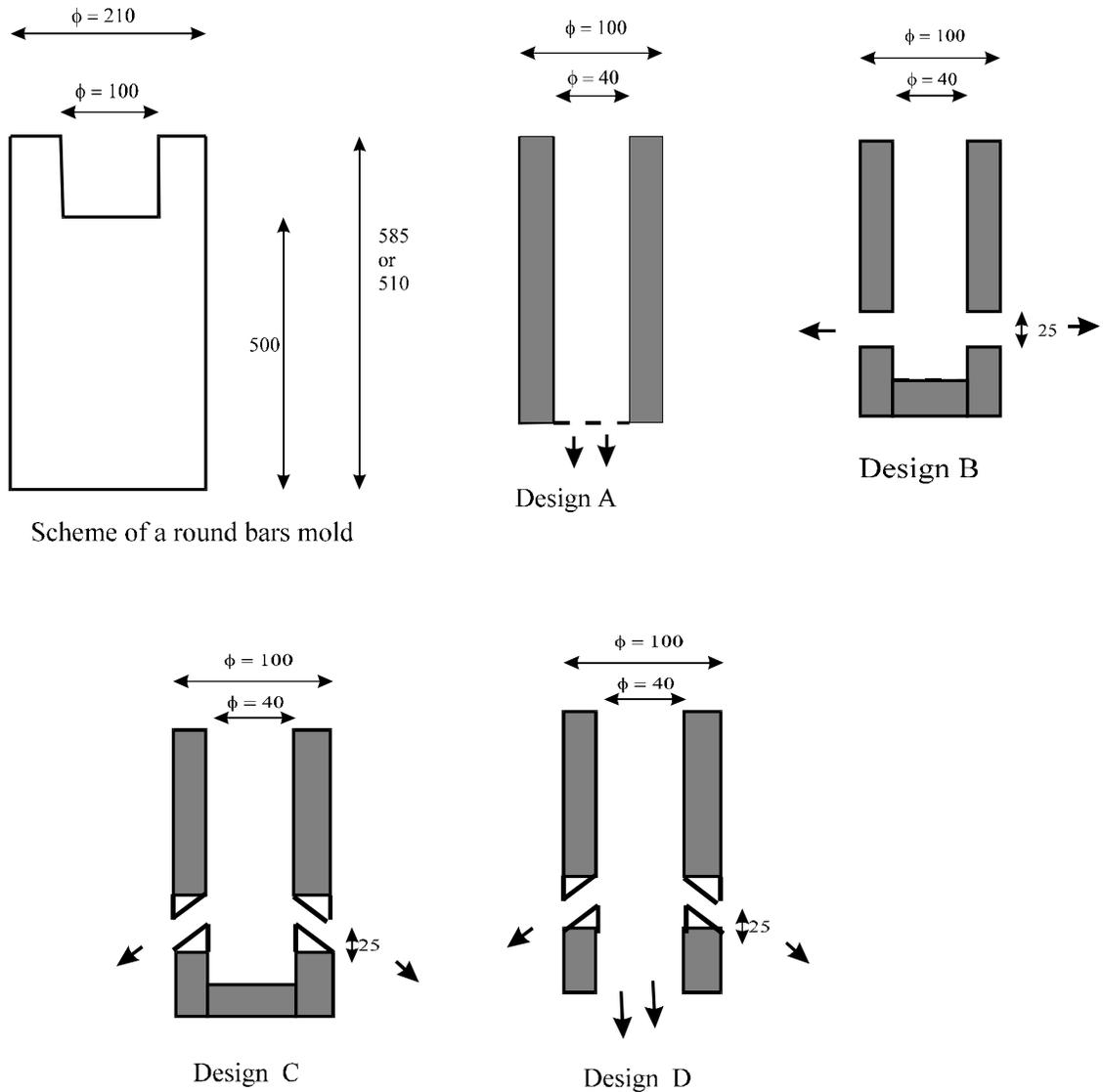


Figure 2 .- Schem of the SENs (Not to scale)

In figure 3, the velocity map and turbulence intensity ($It = \sqrt{k}$) map obtained for the C-design are shown (casting speed = $1.4m/min$). The boundary conditions are:

[*Inflow of liquid steel (e)*

v_e^n : the normal velocity depends on the continuous casting speed

$v_e^t = 0$; tangencial velocity

$k_e = 0.01 (v_e^n)^2$, Reference^{7,20}

$\epsilon_e = k_e^{3/2} / (d_b/2)$

[*Mold wall*

The wall function, $\Delta = 5 \text{ mm}$

[*Free surface of the steel, meniscus, (s)*

$v_s^n = 0$, normal velocity to the surface

$$\frac{\partial v_s^t}{\partial n} = \frac{\partial k_s}{\partial n} = \frac{\partial \epsilon_s}{\partial n} = 0$$

[*Mold exit (o)*

$v_o^t = 0$, parallel exit flow

$$\frac{\partial v_o^n}{\partial n} = \frac{\partial k_o}{\partial n} = \frac{\partial \epsilon_o}{\partial n} = 0$$

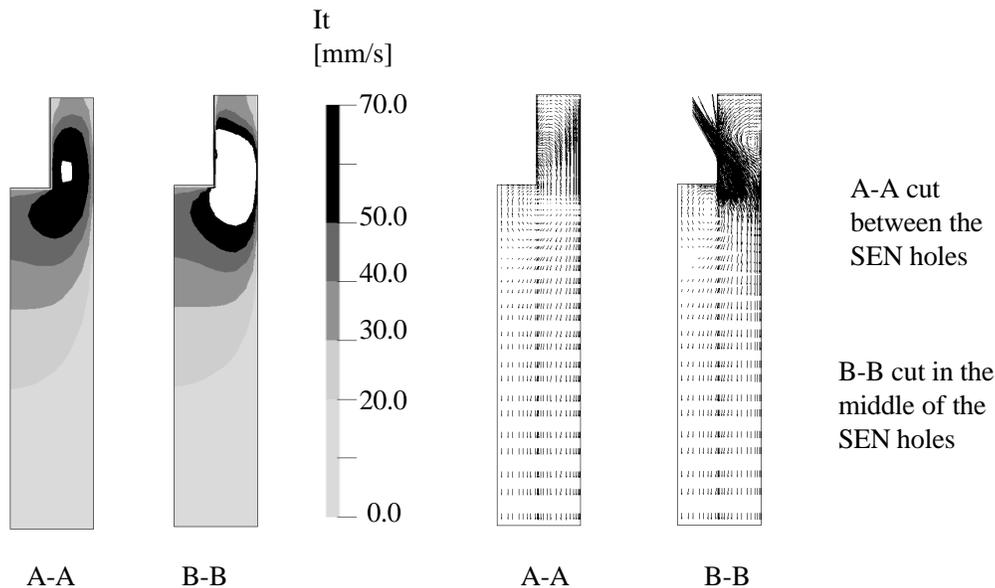


Figure 3 .- Numerical results of the SEN - Design C

The numerical results provide information on:

- flow and mixing intensity at the meniscus,
- flow and mixing intensity at the locations inside the mold stagnation zones,
- flow friction on the mold wall.

The effect of different operational variables was also analyzed for some of the SEN designs, among them:

(1) *Liquid steel level in the mold*

In case of using direct out-flow nozzles (design A) the higher the liquid steel in the mold the bigger the dead zone in the meniscus.

In case of using lateral out-flow nozzles (designs B, C, D) the recirculation flow in the meniscus zone is bigger.

(2) *Casting speed*

The continuous casting speed can change approximately a 50 %. Then, the flow presents a similar behaviour except for the velocity module.

(3) *Turbulence intensity in the inflow*

The dimension of the inflow jet in the mold increases as the turbulence intensity increases.

SIDERCA continuous casting process engineers built a 1:1 water model of the round mold, including the SEN.

In figure 4 we show the numerical results and experimental results when using an injection of die tracer ($KMnO_4$) into the inlet water stream for the design A.

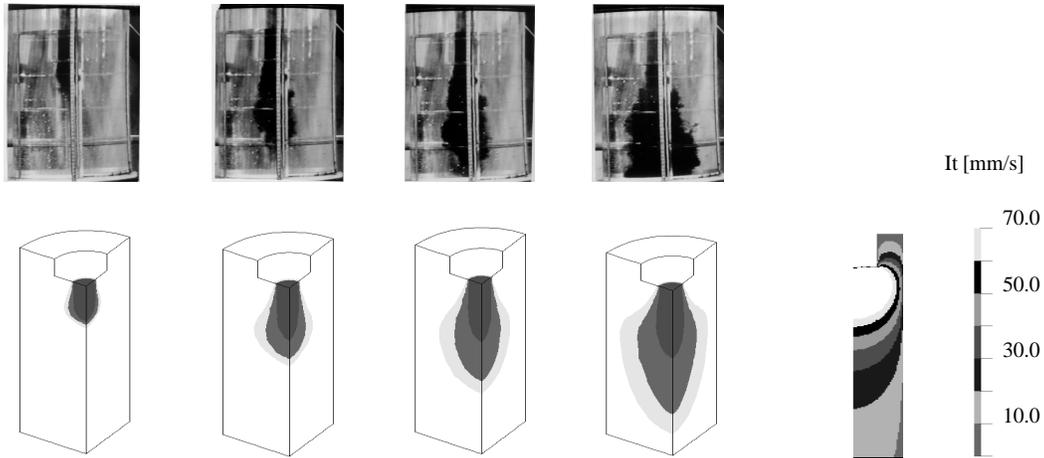


Figure 4.- A-Design SENs

In figure 5 we show the velocity and turbulence intensity distribution inside the SEN in the round bars continuous casting at TAMSA (Veracruz, Mexico).

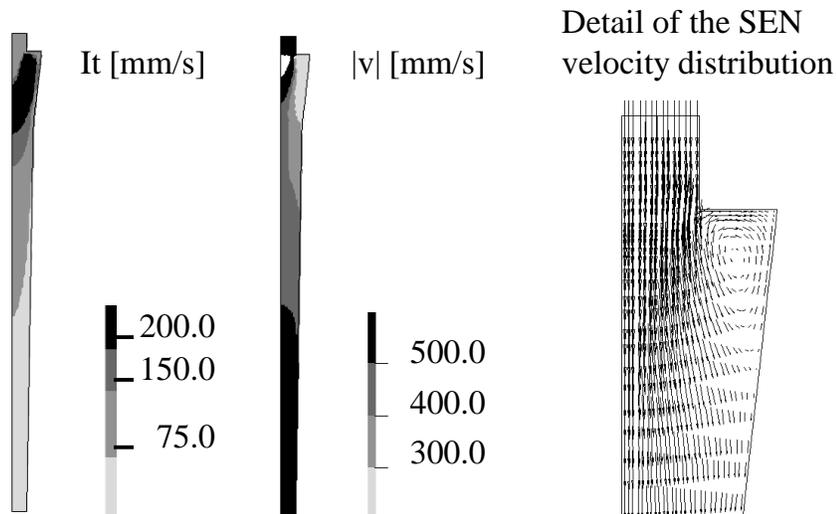


Figure 5.- Inside of the SEN (design A)

The step inside the nozzle induces a recirculation zone that can lead to “clogging”⁴⁰.

4.1.2. Slabs continuous casting

In general, the nozzles for slab mold have a lateral out-flow. Figure 6 shows the model domain (stopper rod / nozzle). As the stopper rod is centered in the cavity an approximately axisymmetric flow is obtained. The highest turbulence intensity is developed at the entrance of the ring space (stopper rod / nozzle). A fully developed flow was reached at 300 mm from the stopper rod. This fact will allow the detailed study of the stopper rod and the nozzle flow separately.

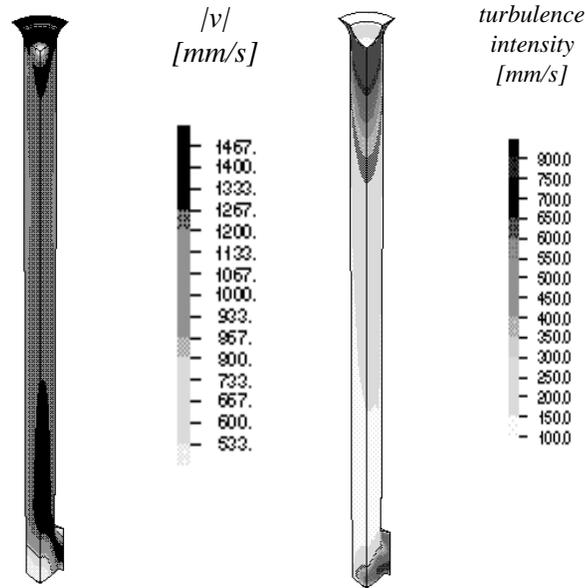


Figure 6 - Stopper rod and SEN for slab mold

As recirculation occurs in the out-flow walls of the nozzle. It is not possible to have a complete description of the flow using ad-hoc 2D models.

In Figure 7 it can be seen that the jet does not follow exactly the inclination of the outflow so it is interesting to know the real values of the angles and of the jet velocities. The following weighted averages have been calculated¹¹:

$$\langle V_i \rangle = \frac{\sum_e V_i^e |V_n^e| A^e}{\sum_e |V_n^e| A^e} \quad ; \quad \langle \theta \rangle = \arctg \left(\frac{\langle V_z \rangle}{\sqrt{\langle V_x \rangle^2 + \langle V_y \rangle^2}} \right) \quad ; \quad \langle I_t \rangle = \frac{\sum_e I_t^e |V_n^e| A^e}{\sum_e |V_n^e| A^e} \quad ;$$

$$\beta = \frac{\sum_e A^e}{A_T}$$

where:

$\langle V_i \rangle$: mean jet velocity in the x , y and z directions;

$|V_n^e|$: absolute value of the mean normal velocity;

$\langle \theta \rangle$: mean jet angle;

$\langle I_t \rangle$: mean jet turbulence intensity;

- β : fraction of the lateral outlet area;
 V_i^e : mean jet speed in the element e in the x , y and z directions;
 I_t^e : mean turbulence intensity in the element e ;
 A^e : area of the element e in the lateral outlet;
 A_T : modelled total area of the lateral outlet;
 \sum_e : it shows an addition of the lateral outlet elements where there is liquid steel outflow (it does not include the recirculation elements of the lateral outlet).

In Figure 7 we show the detail of the SEN for a medium rate flow, $Q_{mid} = 4.01 \cdot 10^6 \text{ mm}^3/\text{s}$. Approximately the 66 % of the liquid steel goes out through the 33 % of the available outlet area; the other 34 % of the liquid steel goes out through the 42 % of the outlet area of the nozzle and in the remaining 25 % of the area there is a recirculating flow.

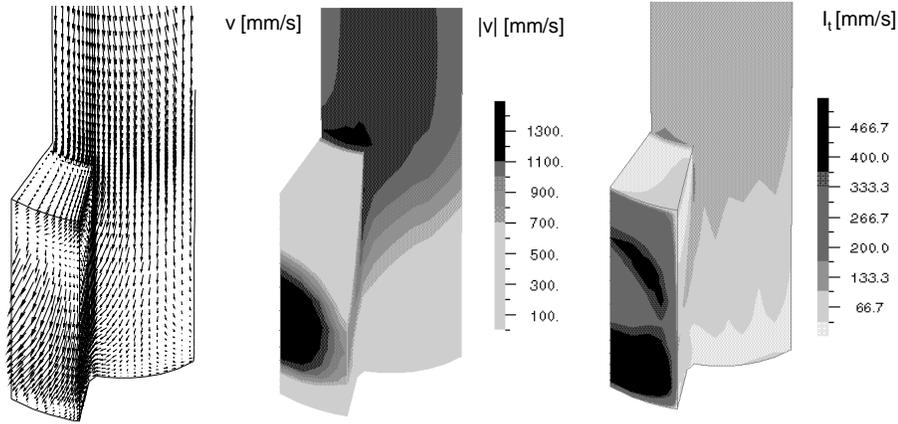


Figure 7 - Detail of the lateral SEN

The Table 2 show the numerical results at the port outlet for different operating conditions. The results of the mean values shows:

- The mean jet angle is not function of the rate flow.
- The mean jet turbulence intensity increase when the rate flow increase.
- The fraction of the lateral outlet area does not change with the rate flow.
- The relation between the turbulence intensity and velocity at the outlet port does not change with the rate flow.

Rate flow	V_c	w	$\langle V_x \rangle$	$\langle V_y \rangle$	$\langle V_z \rangle$	$\langle V \rangle$	$\langle \theta \rangle$	$\langle I_t \rangle$	β	$\langle I_t \rangle / \langle V \rangle$
[mm^3/s]	[m/min]	[mm]	[mm/s]	[mm/s]	[mm/s]	[mm/s]		[mm/s]		
$2.52 \cdot 10^6$	1.05	780	592	99	-397	719	33.5	217	0.73	0.30
$4.01 \cdot 10^6$	1.05	1240	925	159	-623	1127	33.6	368	0.75	0.33
$7.29 \cdot 10^6$	1.45	1630	1687	294	-1119	2046	33.2	658	0.75	0.32

Table 2 - Calculated values at the port outlet

In figure 8 we present a lateral SEN design used at SIDERAR (San Nicolas, Argentina) in its slabs continuous casting installation for two mold widths¹³.

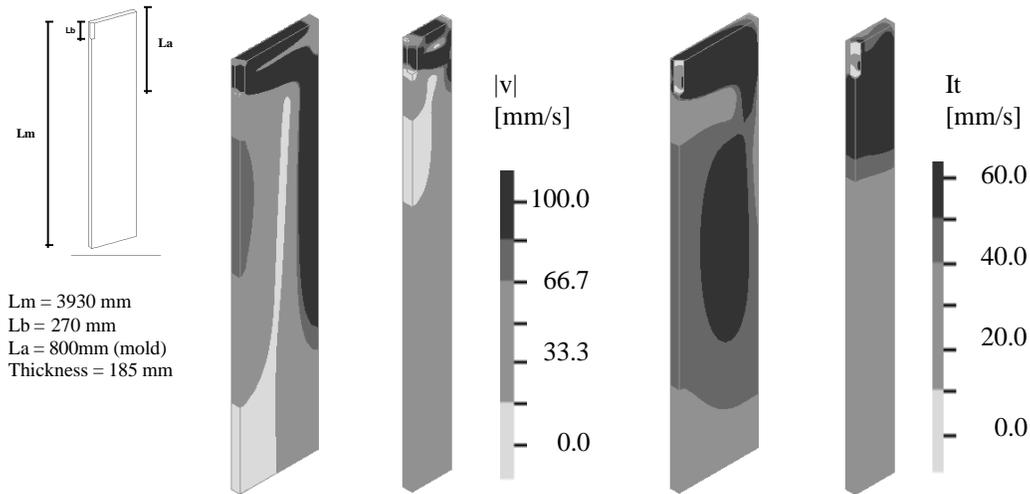


Figure 8.- Slabs continuous casting

From Fig. 8:

- [The narrower mold has a shortest flow development
- [There is a higher turbulence intensity inside the mold when the width is smaller.
- [Areas of low liquid steel movement are observed in the mold center for maximum width molds.

4.2. Tundish

The tundish^{5,8,9} is a refractory vessel placed between the ladle and the mold to supply and distribute the liquid steel to different SENs.

In the tundish the non-metallic inclusions have to float to the free surface and be trapped by the slag layer. The jet of liquid metal from the ladle to the tundish generates a high turbulence zone that induces a possible oxidation of the liquid steel.

Therefore, the velocity distribution inside the tundish has a strong influence on the removal of inclusion particles and the re-oxidation of liquid steel.

In the present paper we study two different design of flow control devices:

- the baffle with holes
- the impact pad

In figure 9 we show the numerical results of turbulence intensity for the two devices. In both cases a 2D simetric approximations was made.

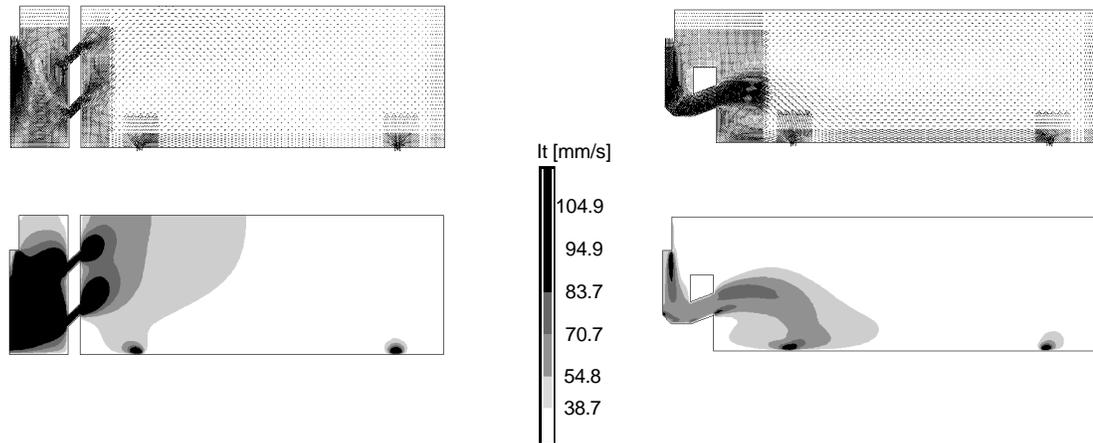


Figure 9 .- Velocity and turbulence intensity distribution in the tundish

From results in Fig. 9 it can be seen that when using the impact pad the liquid steel free surface has less turbulence; this is a favorable feature for the slag inclusions removing function

5. CONCLUSION

The iterative algorithm ($k-L$)-predictor / (ϵ)-corrector scheme for solving turbulent flows^{1,4} was applied to the analysis of different continuous casting processes.

In this paper three application examples were shown:

1. The analysis of different SEN options in the mold of a round bars continuous casting mold.
2. The analysis of one SEN design operating with different mold sizes in a slabs continuous casting.
3. The tundish of a continuous casting installation.

These examples, shown that the algorithm developed can be used to obtain solutions of $k-\epsilon$ turbulent model equations even in 3D complex geometry situations.

The numerical model is used as an engineering tool for understanding and optimizing the steelmaking processes.

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