A NUMERICAL MODEL FOR MULTIPHASE FLOW ON OIL PRODUCTION WELLS

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Abstract

A numerical model for the analysis of multiphase flow on vertical or slightly inclined wells has been developed. The model calculates flow properties (velocity of each phase, volumetric fraction of each phase, pressure and fluids properties) on gas-oil-water wells as function of depth. Fluids properties are obtained under the assumption of black oil model by means of correlations taken from literature, requiring only petroleum °API and the gas specific gravity as input data.

The model may be applied to simulate both liquid flow and gas-liquid flow. In this case, different flow patterns are taken into account: -bubble, slug, dispersed bubble and annular- depending on flow conditions, which are determined from fluid properties and production rates of oil, gas and water. Flow in tubings consisting of several sections with different diameters and inclinations may also be simulated.

The model was validated by comparisons of measured and calculated the pressure variation along the well Good agreement was found between the numerically predicted pressure drop and measurements taken from different databases from open literature. As a consequence the proposed model proves to be a reliable tool to describe the flow on oil-gas-water wells.

The developed numerical model takes into account the most relevant effects that take place in a production well including multiphase flow, presence of different flow pattern, mass transfer from gaseous to liquid phase and influence of gas-liquid flow pattern on wall friction. Special attention is paid to the velocity profile of each phase along the well. Ishii's model for two-fluid flow is used to prescribe the slip velocity between liquid and gaseous phases and to determine the acceleration term contribution to the pressure gradient. This model is actually being employed for corrosion rate calculations inside production wells.

1. Introduction

The study of the multiphase flows (water – oil – gas) is of major importance in oil industry since it is found quite frequently during the production process. The physics involved in these flows is very complex due to interactions between the different phases. In order to deal with this complexity, sophisticated numerical models with several parameters (most of them determined from experiments) are required.

The complexity of the problem leads to a number of simplifying assumptions and to the use of correlations to model some terms of the equations. Many numerical methods have been proposed in order to prescribe flow variables (velocities of each phase, volume fraction of each phase, flow pattern, pressure gradient) along the tubing for vertical upward flow. There is a wide variety of numerical methods, including simple models where liquid and gas are supposed to have same velocity [1-3], models that account for slippage between gas and liquid but do not consider the existence of different flow patterns [4-6], models that take into account different flow patterns [7-12] to complex mechanistic models [13-17].

In this work an alternative numerical model to estimate the flow characteristics along a vertical or near vertical pipe is presented. The proposed method belongs to the class of models described in references [7-12]. However, instead of using a correlation for liquid hold up we use a correlation for the slip velocity between liquid and gaseous phases and calculate the hold up from conservation equations. It was codified in a FORTRAN code named *GOWflow*.

In the next section the general equations of the model are introduced. In section 3 the modeling of different terms taking part in the equations is presented. The algorithm is described in section 4. Validation against measurements of pressure drop is presented in section 5. Examples of application are shown in section 6. Last section is devoted to conclusions and future work.

2. Equations

Governing equations were obtained from mass conservation for each component and global momentum conservation principles in steady state [18]. The equations were averaged across the –assumed circular– section *S* of the pipe in order to obtain a one-dimensional model.

As a consequence a system of ordinary differential equations is obtained for the section averaged values of pressure (*p*), gas velocity ($\tilde{v}_G$), liquid velocity (assumed the same for oil and water, $\tilde{v}_L$), gas volume fraction (α_G), oil volume fraction (α_O) and water volume fraction (α_W). These variables are only function of the axial coordinate, *z*.

$$S\alpha_O \tilde{v}_L \rho_O - R_S q_O \rho_O^{SC} = q_O \rho_O^{SC} \tag{1}$$

$$S\alpha_G \tilde{v}_G \rho_G + R_S q_O \rho_G^{SC} + R_{WS} q_W \rho_G^{SC} = q_G \rho_G^{SC} \tag{2}$$

$$S\alpha_W \tilde{v}_L \rho_W - R_{WS} q_W \rho_G^{SC} = q_W \rho_W^{SC} \tag{3}$$

$$\frac{dp}{dz} = \rho g_z + \frac{d\tau_{zz}}{dz} + \frac{2}{R}\tau_{rz}(R) - \frac{d}{dz}\left(\alpha_L \rho_L \tilde{v}_L^{\,2} + \alpha_G \rho_G \tilde{v}_G^{\,2} + CT\right) \tag{4}$$

$$\alpha_O + \alpha_G + \alpha_W = 1 \tag{5}$$

$$V_d = (1 - \alpha_G)\,(\tilde{v}_G - \tilde{v}_L) \tag{6}$$

In the above equation the following variables were introduced

$\rho_O,\ \rho_G,\ \rho_W$	are the densities of oil, gas and water,
$\rho_L\ ,\rho$	are the liquid and total density $\alpha_L \rho_L = \alpha_O \rho_O + \alpha_W \rho_W$; $\rho = \alpha_L \rho_L + \alpha_G \rho_G$,
$R_S,\ R_{WS}$	are gas - oil and gas - water solution ratios,
$q_O,\ q_G,\ q_W$	are production rates of oil, gas and water,
R	is the tube radius,
τ_{zz}, τ_{rz}	are components of the deviatoric strain tensor,
α_L	is the liquid volumetric fraction $\alpha_L = \alpha_O + \alpha_W$, and
g_z	is the component of gravity acceleration along the tube.

and local variables $\hat{x}_i$ were averaged using either $x_i = \int_S \hat{x}_i \; dS / S$ (for $x_i = p,\ \alpha_G,\ \alpha_O,\ \alpha_W$) or $\tilde{x}_i = \int_S \hat{\alpha}_i \; \hat{x}_i \; dS / \int_S \hat{\alpha}_i \; dS$ (for $\tilde{x}_i = \tilde{v}_L, \tilde{v}_G$).

The main characteristic of the present model is the absence of a correlation for the liquid hold up α_L and the use of a correlation for drift velocity V_d instead (see for instance [19]). Focus is made on the acceleration term in equation (4), including the covariant term, CT, that arises from the averaging process.

The system is closed by prescribing models for fluid properties, for covariant terms, CT, for the gas drift velocity V_d, and for the viscous terms, $d\tau_{zz}/dz + 2\tau_{rz}(R)/R$.

3. Modeling

As stated in the previous paragraph, in order to solve the system of equations (1-6) some terms need to be modeled. Some of the modeled terms depend not only on the local value of the variables but also on the type of flow pattern present. So, a criterion for flow pattern recognition should be established.

3.1 Flow pattern

Drift velocity and viscous forces depend not only on the fluid properties but also on the type of flow present in the pipe. Four different two-phase flow patterns are generally recognized on liquid-gas flows on vertical ducts (see for instance Bertuzzi et al. [20])

- Bubble flow: The liquid is in continuous phase with small bubbles of gas.
- Slug flow: Now the gas bubbles are large and elongated and bubbles separated by slugs of liquid.
- Churn flow: The bubbles disappear. The continuous phase is alternatively liquid or gaseous.
- Annular flow. A continuous gas phase with the liquid as a film wetting pipe walls and droplets in the gas core.

Figure 1 shows a schematic representation of each of these flows.

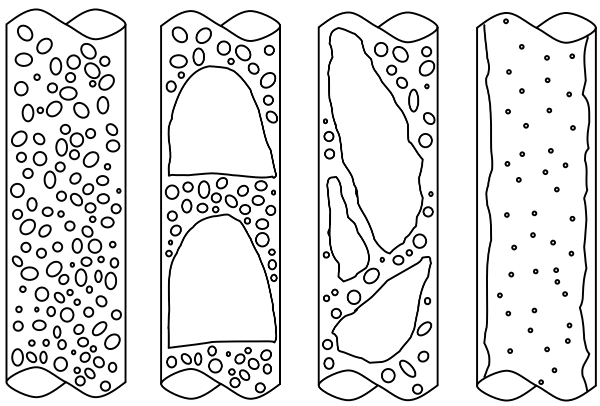

Figure 1: Different flow patterns on a vertical tube

Along this work Ishii's transition criteria [18] et al. will be used to identify the flow pattern for given gas and liquid velocities and fractions. The model considers the above mentioned flow patterns for two phase flow and single phase liquid flow (single phase gaseous flow requires a compositional model)

3.2 Fluid properties

In order to calculate physical properties of the fluids like the densities of each phase and the solution ratios, a black oil model is adopted.

The black oil model assumes that the gas-oil mixture to be analyzed can be described in terms of two single parameters: the oil API gravity and the gas specific gravity. Fluid properties at a given point in the well may be obtained from these parameters and section averaged values of pressure and temperature. Pressure is obtained from the momentum conservation equation (4) while the temperature is supposed to be a known function of the vertical coordinate; A linear dependence of temperature with depth is assumed with a constant gradient of 1.5 °F /feet [21]. This gradient is modified to fit the bottom hole temperature, when it is available.

Correlations for the different fluid properties were obtained from open literature. The following correlations were considered in our model,

- Solution gas-oil ratio, from Standing [22] and Beggs [23].
- Oil formation volume factor, from Standing [22] and Beggs [23].
- Oil isothermal compressibility, from Vasquez and Beggs [24].
- Oil viscosity, from Beggs and Robinson [25], and Vasquez and Beggs [24].
- Solution gas-water ratio, from Ahmed [26].
- Water formation volume factor, from Gould [27].
- Water isothermal compressibility, from Meehan [28].
- Water viscosity, from van Wingen [29]
- Surface tension, from Baker and Swerdloff [30].
- Gas Viscosity, from Lee, Gonzalez and Eakin [31].

Densities of liquid phases are calculated from the corresponding formation volume factors, gas in solution ratios and isothermal compressibility. Gas density is obtained from a real gas equation of state.

All these relation depend strongly on pressure, temperature and the black oil parameters (i.e. the oil °API and the gas specific gravity)

3.3 Covariant term

From the section averaging process a covariant term of the form

$$CT = \int_S \left(\hat{\alpha}_G \hat{\rho}_G \hat{v}_G (\hat{v}_G - \tilde{v}_G) + \hat{\alpha}_L \hat{\rho}_L \hat{v}_L (\hat{v}_L - \tilde{v}_L) \right) dS / S$$

appears in the moment conservation equation. The following correlation proposed by Ishii [18] is used to model this term.

$$CT = 0.3\left(1 - \sqrt{\rho_G / \rho_L}\right)\left[1 - \exp\left(-18\, \alpha_G\, \alpha_L\right)\right]\left(\rho v^2 + \frac{\rho_L\, \rho_G}{\rho} \frac{\alpha_G}{\alpha_L} V_d^{\,2}\right)$$

3.4 Viscous terms

Internal stress are supposed to have a negligible influence on the moment conservation equation, so the viscous terms reduce to the wall friction contribution, i.e., $d\tau_{zz}/dz + 2\tau_{rz}(R)/R \approx 2\tau_{rz}(R)/R$. This term –for each flow regime– was modeled using Kabir and Hasan correlations [32].

- Only Liquid/Bubbly flow $\dfrac{2}{R}\tau_{rz}(R) = -\dfrac{f\,\rho\, v|v|}{4R}$
- Slug/Churn flow $\dfrac{2}{R}\tau_{rz}(R) = -\alpha_L \dfrac{f\,\rho_L\, v|v|}{4R}$
- Annular flow $\dfrac{2}{R}\tau_{rz}(R) = -\dfrac{f_C\,\rho_C\,\tilde{v}_G|\tilde{v}_G|}{4R}$

where,

$$v = \frac{\tilde{v}_G\,\rho_G\,\alpha_G + \tilde{v}_L\,\rho_L\,\alpha_L}{\rho};$$

f is the wall friction factor determined by Reynolds number $\mathrm{Re} = 2v\,R\,\rho_L/\mu_L$ and pipe roughness ε;

$$f_C = 0.079(1 + 75\alpha_L)\left(\frac{\mu_G}{2R\,\alpha_G\,\rho_G\,\tilde{v}_G}\right)^{0.25} \quad ; \quad \rho_C = \frac{\alpha_G\,\rho_G\,\tilde{v}_G + E\,\alpha_L\,\rho_L\,\tilde{v}_L}{\alpha_G\,\tilde{v}_G + \alpha_L\,\tilde{v}_L};$$

$$E = \begin{cases} 0.0055\,x^{2.86} & if\ x < 4 \\ 0.857\,\log(x) - 0.2 & if\ x > 4 \end{cases} \quad with\ x = \alpha_G\,\mu_G\,\tilde{v}_G\sqrt{\frac{\rho_G}{\rho_L}}\sigma \times 10^4;$$

μ_G, μ_L are the viscosities of the gaseous and liquid phases.

3.5 Drift velocity

A model for drift velocity, V_d, is also required. Correlations are provided for the distribution parameter C_0 and the averaged drift velocity $\tilde{V}_d$ which are related to the drift velocity by the expression

$$V_d = (C_0 - 1)(\alpha_G\tilde{v}_G + \alpha_L\tilde{v}_L) + \tilde{V}_d \tag{7}$$

These correlations depend on flow pattern, gas density, liquid density, gas fraction, tube radius and surface tension (σ) and where taken from Ishii's work [18].

- Bubbly flow

$$C_0 = 1 + 0.2\left(1 - \sqrt{\rho_G / \rho_L}\right)\left[1 - \exp\left(-18\alpha_G\right)\right] \quad ; \quad \tilde{V}_d = 1.53\left(g\sigma \frac{\rho_L - \rho_G}{\rho_L}\right)^{1/4}$$

- Slug/churn flow

$$C_0 = 1.2 \quad ; \quad \tilde{V}_d = 0.35\left(2\, g\, R \frac{\rho_L - \rho_G}{\rho_L}\right)^{1/2}$$

- Annular flow

$$C_0 = 1 + \left(1 - \alpha_G\right)\left(\alpha_G + 4\sqrt{\frac{\rho_G}{\rho_L}}\right)^{-1} \quad ; \quad \tilde{V}_d = \left(2\, g\, R \frac{\rho_L - \rho_G}{\rho_L} \frac{1 - \alpha_G}{0.015}\right)^{1/2}\left(1 - \alpha_G\right)\left(\alpha_G + 4\sqrt{\frac{\rho_G}{\rho_L}}\right)^{-1}$$

4. Algorithm

The numerical solution of the system of equations 1-6 starts by solving all variables at the wellhead and progresses downwards point by point along the grid.

The program requires as input data the wellhead conditions (temperature and pressure) the geometry of the well (inner diameter, inclination and length of each segment), the production flow rates (oil production, gas production, water production) and the fluid characteristic parameters (°API, gas specific gravity)

The algorithm comprises an outer loop which covers all possible segments of the piping. For each segment, there is a second loop on the discretization points where the equations are solved. The system is highly nonlinear, so an iterative algorithm was developed (see Figure 2).

Convergence demands a few seconds on a 2GHz Pentium PC, excepting for very high gas flow rates on small section pipes, where more time (up to three or four minutes in a 2GHz Pentium PC) is needed. Numerical results showed no significant modification by mesh refining.

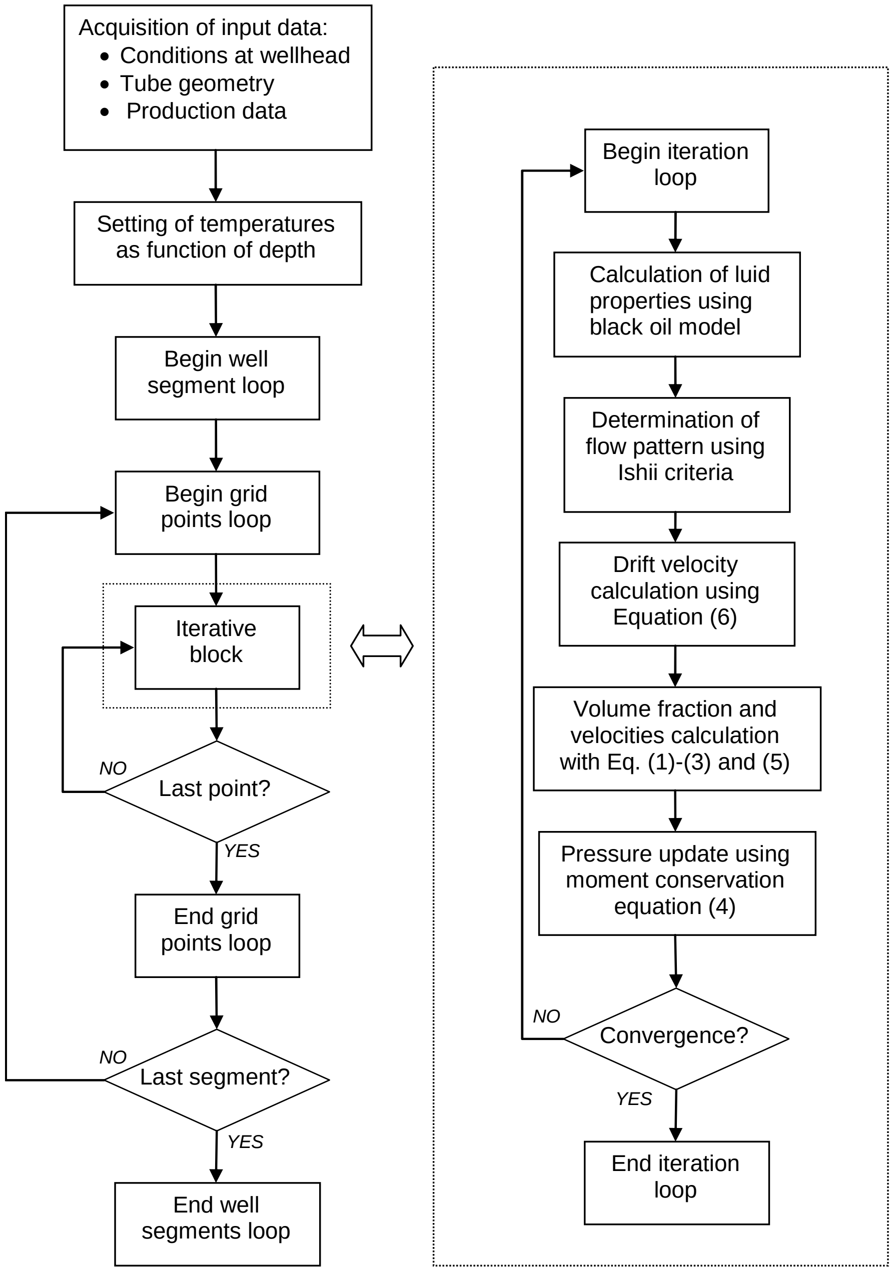

Figure 2: Algorithm flow chart

5. Validation

Validation was carried out by comparison of numerically calculated pressure drops with experimental data from several oil production wells (references [9] and [10]). Results are presented in Figure 3, where it may be observed that most numerical estimations lie within the 15 % error band.

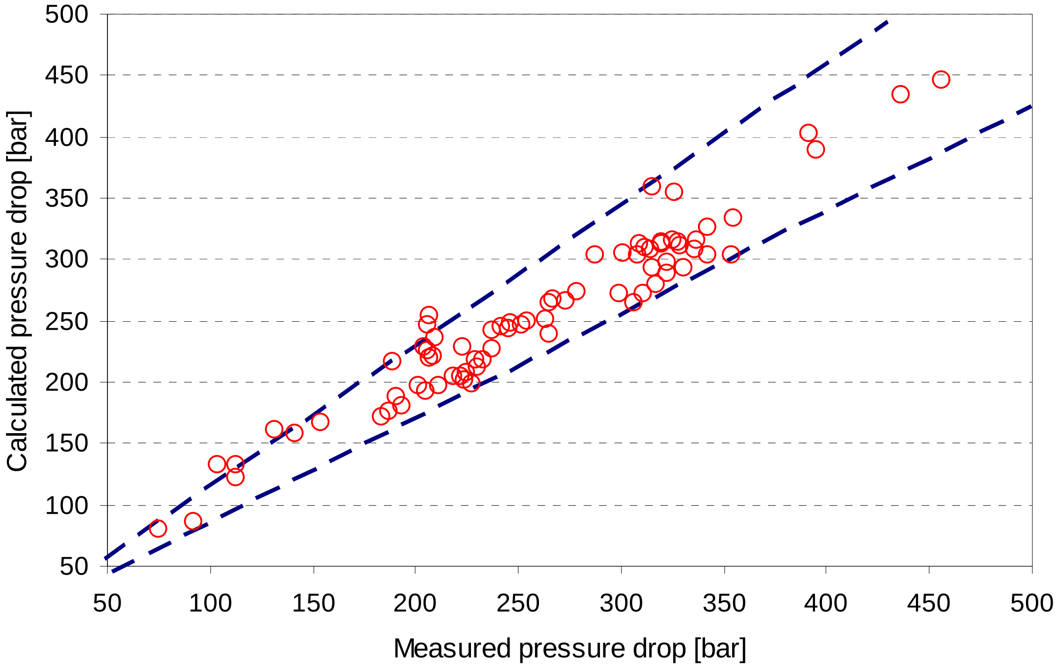

Figure 3: Measured and calculated bottomhole pressure for Aziz et Al [9] and Chierici et al [10] databases. Dotted lines represent the limits of the 15% error band.

The average relative error,

$$\varepsilon = \frac{1}{N}\sum_{1}^{N}\left|\frac{\Delta p_C - \Delta p_M}{\Delta p_M}\right| \times 100$$

(where Δp_C is the calculated pressure difference between bottom hole and wellhead, Δp_M is the measured pressure difference and N is the number of wells considered) is of 9.7% on the 76 analyzed wells (45 taken from Aziz et Al [9] database and 31 from Chierici et Al [10] database).

6. Results

Results obtained with the numerical model are illustrated in Figure 4 and 5. The figures present each phase fraction and velocity for a well producing oil (*11900 bbl/D*), water (*180 bbl/D*) and gas (*8.4 MM scf/D*) . Temperature and pressure conditions at the wellhead are 68 °C and 63 bars respectively.

The tubing has a total length of *2900 m*. It is vertical up to a depth of *1500 m* and the has a 15° inclination. Tube inner diameter is *4.89* inches at the well head but changes to *8.68* inches below *2000 m.* Three sections have to be considered (Figure 4),

- Section I: From the well head to *1500 m*, with *0°* inclination and *4.89* inches diameter.
- Section II: From the *1500 m* to *2000 m*, with *15°* inclination and *4.89* inches diameter.
- Section III: Below *2000 m*, with *15°* inclination and *8.68* inches diameter.

The flow is not significantly affected by the (slight) change in inclination but a discontinuity in the velocity is found for both phases at the point where the diameter changes due to mass conservation principle.

The bubble point is located at *2700 m* from the well head, where the fluid becomes single phase (liquid). A slug-bubble flow pattern transition takes place at *1000 m* below the wellhead (Figure 5). The drift velocity increases significantly when passing from bubble flow to slug flow, thus a "jump" may be observed in all variables in order to take this discontinuity into account.

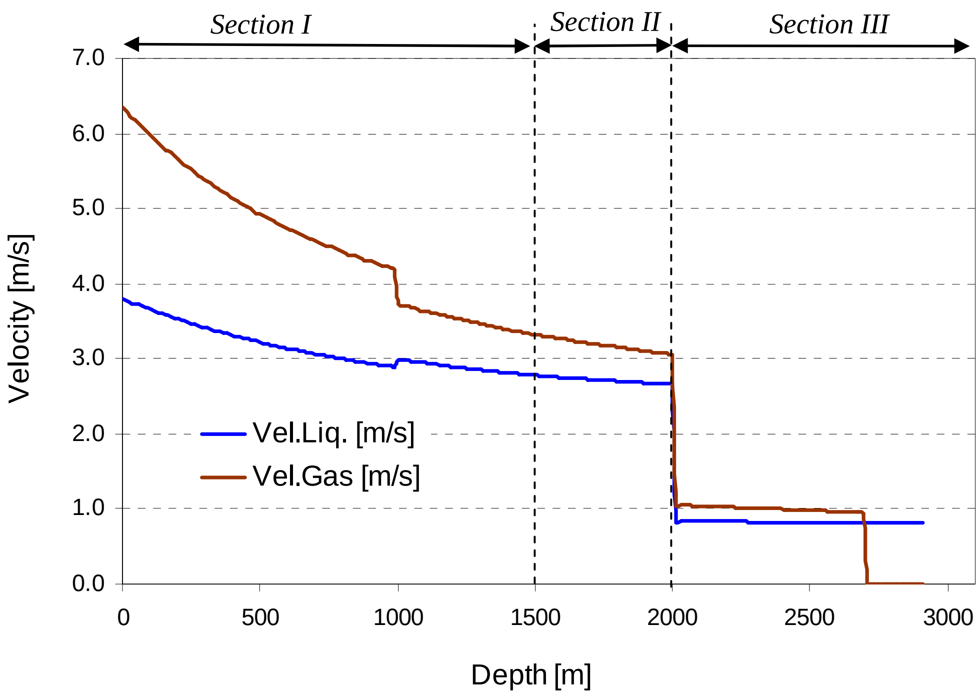

Figure 4: Oil velocity and Gas velocity along a vertical well.

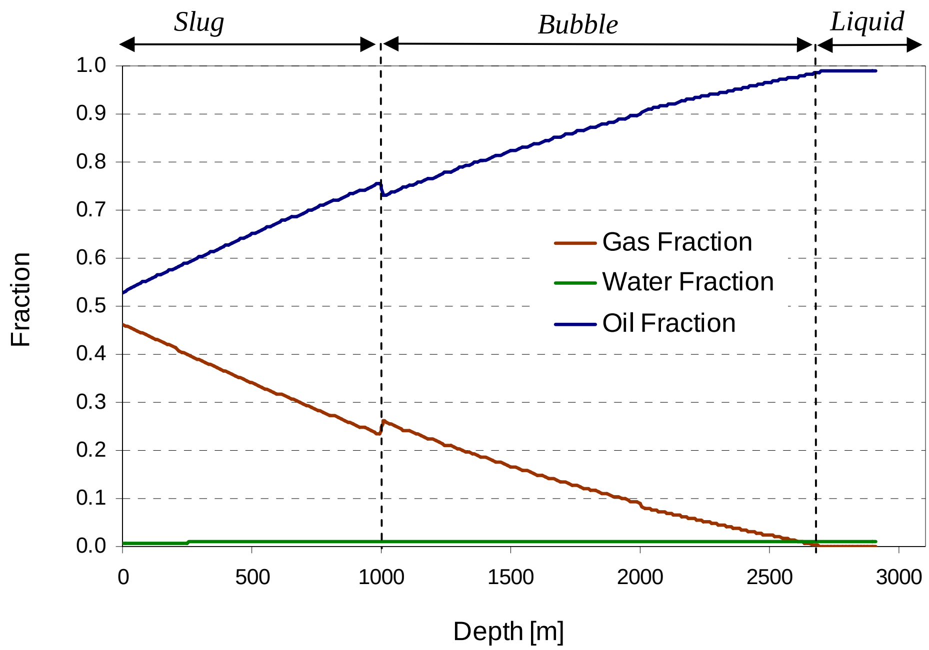

Figure 5: Gas fraction, oil fraction and water fraction along a vertical well.

7. Conclusions and future work

A numerical model able to simulate the three phase (gas-oil-water) flow along a vertical (or near vertical) production well has been developed. Section averaged values of flow variables are calculated as a function of depth taking the most significant effects as mass transfer, wall friction and different flow patterns (liquid – bubble – slug/transition – mist) into account. Well composed by several section of different diameters and inclinations may also be analyzed. Results can be obtained in very short period of time on a personal computer. Unlike most existing methods, the model focuses in the calculation of the acceleration term in the momentum conservation equation. A drift velocity correlation is used to close the model instead of a liquid hold up correlation

The analysis of several oil wells reported in literature showed that error dispersion in the calculation of bottom hole pressure is similar to that obtained by models found in literature. The model showed itself robust enough to represent correctly the flow under rather exigent conditions. It proved to be stable for a wide range of well geometries, production conditions and fluid properties.

The actual version of the model is adequate to simulate the flow in vertical (or near vertical) wells for non volatile oils. In order to analyze horizontal flow the program is being modified to include flow patterns present in such flows. Also, a compositional model is being developed for the analysis of volatile oils.

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