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A new shell element for elasto-plastic finite strain analysis. Application to the collapse and post-collapse analysis of marine pipelines

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Abstract

The infinitesimal strain version of the MITC4 shell element (Dvorkin and Bathe [6]) was previously successfully used for the analysis of deep-water applications of steel marine pipelines. The collapse and post collapse responses were modeled and compared with experimental results (Toscano et al. [12]). Even though in those verifications the matching between numerical and experimental results was excellent, it was also noticed that in the post–collapse regime very high strains are developed in the steel shell. In Ref. [12] a version of the MITC4 that uses a posteriori updates of the shell thickness was used to incorporate into the model the finite strain behavior. The results indicated that even though the consideration of finite strains improves the solution, the room for improvement - when the overall equilibrium paths are considered - is so small that it hardly justifies the use of a more expensive numerical model. However, if local strains are sought, the finite strain model produces much better approximations to the actual situation. Hence, the motivation for shell element formulations apt for finite strain elasto-plastic analyses is still opened.

In previous publications we presented a new shell element formulation, the MITC4-3D that we developed for finite strain analysis (Toscano and Dvorkin [13, 14]) using the MITC4 strains interpolation (Dvorkin and Bathe [6]) and 3D constitutive relations.

In this paper we are going to discuss the basic features of the MITC4-3D element and present further verification / validation.

1. Introduction

In 1970, Ahmad, Irons and Zienkiewicz [1] presented a shell element formulation that after many years still constitutes the basis for modern finite element analysis of shell structures. Even tough the A-I-Z shell element was a breakthrough in the field of finite element analysis of shell structures, under the constraint of the infinitesimal strains, it suffers from the locking phenomenon and much research effort has been devoted to the development of A-I-Z type elements that do not present this problem (Bathe [2], Chapelle and Bathe [3], Zienkiewicz and Taylor [15],).

The MITC4 shell element (Dvorkin and Bathe [6]), which was developed to overcome the locking problem of the A-I-Z shell elements has become, since its development in the early eighties, the standard shell element for many finite element codes. However, the limitation of infinitesimal strains is still present in the MITC4 formulation.

In 1995 Dvorkin, Pantuso and Repetto [7] developed the MITC4-TLH element, that based on the original MITC4 formulation can model finite strain elasto-plastic deformations. This element imposes the condition of zero transversal stresses and its computational cost is rather high.

In the present paper we present an element that is also based on the MITC4 formulation and can efficiently model finite strain deformations using a general 3D elasto-plastic material model.

The most relevant differences with the original MITC4 formulation are:

- For each quadrilateral element we have 22 d.o.f.: 5 generalized displacements per node plus 2 extra d.o.f. to incorporate the through-the-thickness stretching.
- We use a general 3D constitutive relation instead of the original laminae plane stress constitutive relation.

2. The MITC4-3D formulation

Some of the basic features of our MITC4-3D element are:

- The shell geometry is interpolated using mid-surface nodes and director vectors.
- The nodal displacements and transverse shear strains are interpolated using the original MITC4 formulation (Dvorkin and Bathe [6]).
- For interpolating the director vectors special care is taken to avoid spurious director vector stretches (Gebhardt and Schweizerhof [8], Simo *et al*. [9-11]).
- Two additional degrees of freedom are considered to include a linear thickness stretching. These thickness-stretching degrees of freedom are condensed at the element level.

2.1 Shell element geometry

Following the MITC4 formulation we define, in the reference configuration, nodes on the shell mid-surface and at each node we define a director vector which represents, at that node, an approximation to the shell mid-surface (Dvorkin and Bathe [6], Simo *et al*. [9-11]).

Therefore, defining inside the element the natural coordinate system (r,s,t) (Bathe [2]), for an element with constant thickness, we can write,

$$ {}^{\tau}\underline{x}(r,s,t) = h_k(r,s)\cdot {}^{\tau}\underline{x}_k + \frac{t}{2}\cdot\left({}^{\tau}\lambda_0 + {}^{\tau}\lambda_1\cdot t\right)\cdot\frac{h_k(r,s)\cdot {}^{\tau}\underline{V}_n^k}{\left\| h_k(r,s)\cdot {}^{\tau}\underline{V}_n^k \right\|}\cdot a \qquad (1) $$

Where,

$h_k(r,s)$: isoparametric 2D interpolation functions (Bathe [2]),

${}^{\tau}\underline{x}$: k-node position vector,

a : constant element thickness,

${}^{\tau}\underline{V}_n^k$: k-node director vetor; with $\left\| {}^{\tau}\underline{V}_n^k \right\| = 1$.

In Eqn. (1) ${}^{\tau}\lambda_0$ is a constant thickness stretching and ${}^{\tau}\lambda_1$ is the through-the-thickness stretching gradient. In our formulation the element ${}^{\tau}\lambda_0$ and ${}^{\tau}\lambda_1$ are discontinuous across element boundaries and they will be condensed at the element level.

2.2 Incremental displacements

The incremental displacements to evolve from the τ-configuration to the τ+Δτ-configuration are,

$$ \underline{u}(r,s,t) = h_k(r,s)\cdot\underline{u}_k + \frac{t}{2}\cdot\left({}^{\tau}\lambda_0 + \Delta\lambda_0 + {}^{\tau}\lambda_1\cdot t + \Delta\lambda_1\cdot t\right)\cdot\frac{h_k(r,s)\cdot {}^{\tau+\Delta\tau}\underline{V}_n^k}{\left\| h_k(r,s)\cdot {}^{\tau+\Delta\tau}\underline{V}_n^k \right\|}\cdot a \qquad (2) $$

For the director vector rotations we can write,

$$ {}^{\tau+\Delta\tau}\underline{V}_n^k = {}^{\tau+\Delta\tau}_{\tau}\underline{\underline{R}}\cdot {}^{\tau}\underline{V}_n^k $$

where ${}^{\tau+\Delta\tau}_{\tau}\underline{\underline{R}}$ is a rotation tensor (Dvorkin *et al*. [5]).

To simplify the formulation, we made the approximation $\left\| h_k(r,s)\cdot {}^{\tau+\Delta\tau}\underline{V}_n^k \right\| = \left\| h_k(r,s)\cdot {}^{\tau}\underline{V}_n^k \right\|$

2.3 Constitutive relation

The shell element formulation developed in this paper is a fully 3D formulation since the in-layer plane stress hypothesis used in the original MITC4 formulation was not invoked in this case. This full 3D constitutive relation is based on:

- Lee´s multiplicative decomposition of the deformation gradient (Figure 1)
- Maximum plastic dissipation
- For the elastic part an hyperelastic relation using Hooke with Hencky strains: ${}^{\tau}_{0}\underline{\underline{\Gamma}} = \underline{\underline{\underline{\underline{C}}}} \cdot {}^{\tau}_{0}H$.

${}^{\tau}_{0}\Gamma^{IJ}$ are calculated as the rotational pull-back of the contravarian components of the Kirchhoff stress tensor. For an ***isotropic*** material, the second order tensor ${}^{\tau}_{0}\underline{\underline{\Gamma}}$ is the stress measure energy-conjugate to the Hencky strain tensor (Dvorkin and Goldschmit [7]).

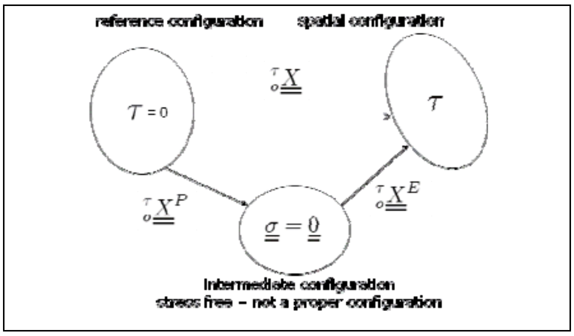

Figure 1: Lee’s multiplicative decomposition

3. The incremental formulation

Using a total Lagrangian formulation we can write the Principle of Virtual Work for the equilibrium configuration at τ+Δτ (Bathe [2]),

$$\int_{{}^{0}V} {}^{t+\Delta\tau}\underline{\underline{\Gamma}} : \delta \underline{\underline{H}}^{E} \cdot {}^{0}dV = {}^{t+\Delta\tau}\underline{\Re}$$

where ${}^{t+\Delta\tau}\underline{\Re}$ is the virtual work of the external loads acting on the solid body in the τ+Δτ-configuration and $\underline{\underline{H}}^{E}$ is an elastic Hencky strain tensor.

The resulting stiffness matrices are, of course, symmetric.

4. Numerical results

4.1 Infinitely long cylinder under internal pressure

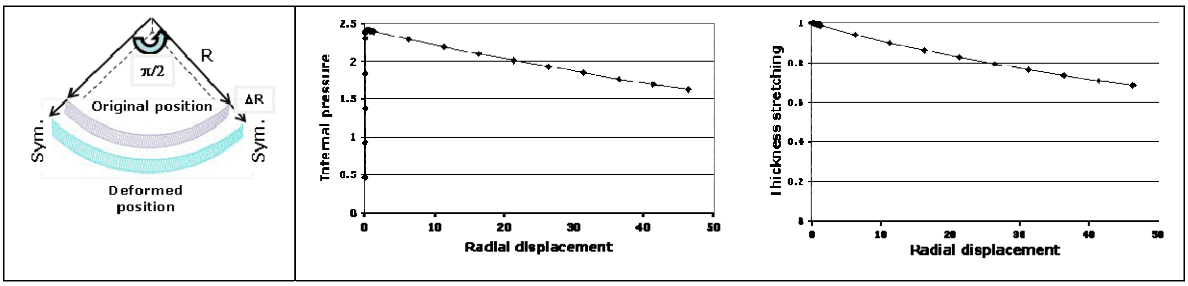

Figure 2: Inflation of an Infinitely long elasto-plastic cylinder

We consider the infinite cylinder represented in Fig. 2 under internal pressure (R_{int}=100. , E=2.1 10^6, E_t=2.1 10^3, a=0.1, μ=0.3, σ_y=2.1 10^3). In the same figure we represent the equilibrium paths obtained for the infinitely long cylinder as well as the predictions of the through-the-thickness stretching.

The d.o.f. ($\Delta\lambda_0$, $\Delta\lambda_1$) are condensed at the element level and (20x20) element stiffness matrices are obtained and assembled into the global stiffness matrices.

4. Conclusions

On the basis of the MITC4 shell element formulation, we developed the MITC4-3D shell element formulation for finite strain analyses of shell structures using general 3D constitutive models. In this paper the new element was implemented for the analyses of elasto-plastic shell structures and the results indicate that it is a very effective element.

5. References

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