FINITE ELEMENT ANALYSIS OF STEEL ROLLING PROCESSES

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Abstract

The numerical simulation of rolling processes requires the coupling of several models that describe different physical phenomena such as the deformation of the work-piece together with its thermo-metallurgical evolution and the thermal evolution of the rolls together with its mechanical deformation. In this paper, using an Eulerian formulation implemented via the pseudo-concentrations technique, we develop the coupling between the purely mechanical model of the work-piece and the thermo-mechanical model of the rolls. We test the numerical formulation in the analysis of the hot rolling of steel coils and in the analysis of the hot mandrel rolling of seamless steel tubes.

1 Introduction

In the modern steel industry, the tolerances in the dimensions and in the mechanical-metallurgical properties of the rolled products are increasingly becoming more stringent; therefore, the process controls of rolling mills have to be able to guarantee that the required product properties are met with a minimum amount of scrap. The first requirement to develop a tight control of any process is to have an in depth knowledge of the process *technological windows*; that is to say, of the locus in the space of the process control variables, where the products meet the required specifications.

Computational models are nowadays a powerful and reliable tool to simulate different thermo-mechanical-metallurgical processes; hence, they are increasingly being used to investigate the technological windows of different processes in the steel industry such as continuous casting, hot rolling, cold rolling, heat treatment, etc. The numerical modeling of rolling processes requires the coupling of several models that describe different physical phenomena such as the deformation of the work-piece together with its thermo-metallurgical evolution and the thermal evolution of the rolls together with its mechanical deformation, [5] to [7].

In previous publications we presented and Eulerian finite element formulation that we developed to simulate metal forming processes using rigid-viscoplastic material models[3] and [8] to [13]. The formulation, based on the flow formulation [25], was developed using the *pseudo-concentrations technique* introduced by Thompson[22][23]: a fixed mesh is used and material moves inside the mesh. In the second section of this paper we review our finite element formulation and we also discuss some technological applications [9][12].

When developing 3D models for simulating the hot rolling of steel coils it is required to include in those models the rolls deformation because it plays a fundamental role in determining the resulting *profile and flatness of the rolled coils* (see the Appendix for a definition of this terminology), in Fig. 1 we present a scheme indicating the mechanical loads acting on the rolls[1]. In the third section we discuss the coupling of the rigid-viscoplastic model that describes the work-piece deformation to a model that describes the rolls thermal and mechanical deformation [10]; in these section we also present the qualification of the model, comparing its predictions with industrial on-line measurements at the hot rolling steel mill of SIDERAR (San Nicolás, Argentina).

The discussed modeling techniques have been implemented in our finite element code **METFOR** and in the fourth section of this paper we describe a technological application developed using **METFOR** for analyzing a steel coils hot rolling mill.

2 The Eulerian formulation based on the pseudo-concentrations technique

In this section we are going to briefly discuss our Eulerian formulation, developed using Thompson's pseudo-concentrations technique.

The following variables are interpolated inside the elements from their nodal values:

- $\underline{\dot{u}}$: material velocities,
- c : nodal pseudo-concentrations

[1] F_1 and F_2 are forces introduced by a hydraulic system that performs an external bending on the rolls; this device is used to control the plates profile / flatness

$c \geq 0$ indicates that actual material is present at the point,
$c < 0$ indicates that there is no material present at the point.

The mesh is fixed and, knowing the velocity field we can calculate the c-distribution with the following transport equations:

$$\dot{\underline{u}} \cdot \nabla c = 0 \; (\textit{stationary problems}) \tag{1a}$$

$$\frac{\partial c}{\partial t} + \dot{\underline{u}} \cdot \nabla c = 0 \; (\textit{transient problems}) \tag{1b}$$

Some relevant features of our finite element formulation:

a. For modeling the equilibrium equations we use finite elements based on the method of *mixed interpolation of tensorial components* with the incompressibility constraint imposed via an augmented Lagrangian procedure [3][8][11] [26]. For 2D problem we use our QMITC-3F quadrilateral element[8][13] and for 3D problems our H1-3F hexahedral element [3][11].

b. For solving the transport equation we use the SUPG method [17] with standard Q1 elements in the 2D case and standard H1 elements in the 3D case.

c. The equivalent plastic strains are transported using,

$$\dot{\underline{u}} \cdot \nabla \overline{\varepsilon} = \frac{\langle c \rangle}{|c|} \dot{\overline{\varepsilon}} \; (\textit{stationary problems}) \tag{2a}$$

$$\frac{\partial \varepsilon}{\partial t} + \dot{\underline{u}} \cdot \nabla \overline{\varepsilon} = \frac{\langle c \rangle}{|c|} \dot{\overline{\varepsilon}} \; (\textit{transient problems}) \tag{2b}$$

d. A staggered iterative scheme is used to couple the equilibrium equations to the c-transport equations:

- For stationary problems we start the iterative algorithm from a trial c-distribution and zero trial velocities: $\dot{\underline{u}} = \underline{0}$.
- For transient problems ($t \longrightarrow t+\Delta t$) we start the iterative algorithm from the converged solution of the previous step.

1. $l = -1$
2. $l = l + 1$
   (i) $j = 0$; $\dot{\underline{u}}^{(j)} = \dot{\underline{u}}^{(j-1)}$
   (ii) $j = j + 1$
   Solve the work-piece nonlinear equilibrium equations keeping constant the c-distribution and the $\overline{\varepsilon}$-distribution
   $\dot{\underline{u}}^{(j)} = f\,(\dot{\underline{u}}^{(j-1)},\ c^{(l)},\ \overline{\varepsilon}^{(l)})$
   (iii) $IF\ \frac{\left\| \dot{\underline{u}}^{(j)} - \dot{\underline{u}}^{(j-1)} \right\|_2}{\left\| \dot{\underline{u}}^{(j)} \right\|_2} \leq UTOL\ .AND.\ \left\| \dot{\varepsilon}_v \right\|_\infty \leq VTOL$
   $THEN \longrightarrow \dot{\underline{u}}^{(l)} = \dot{\underline{u}}^{(j)}\, GO\ TO\ 3$
   $ELSE \longrightarrow\ GO\ TO\ 2.$(ii)
3. *Calculate the c-distribution and $\overline{\varepsilon}$-distribution*
4. $IF\ l = 0\ GO\ TO\ 2$
   $ELSE \longrightarrow IF \frac{\left\| \dot{\underline{u}}^{(l)} - \dot{\underline{u}}^{(l-1)} \right\|_2}{\left\| \dot{\underline{u}}^{(l)} \right\|_2} \leq UTOL$
   $THEN \longrightarrow CONVERGENCE$
   $ELSE \longrightarrow GO\ TO\ 2$

Box I: Taggered iterative algorithm for coupling the equilibrium equations to the c-transport equations.

In the work-piece deformation model we implemented the unilateral contact constraints between the work-piece and the forming tools (rolls) using c-dependent boundary conditions [13]:

- The velocity normal to the rolls is free for $c < 0$ and is set to zero for $c > 0$.
- When a non-physical traction develops on the node for which the c-dependent boundary conditions have established $\dot{\underline{u}}_n = 0$,we remove the velocity constraint.
- When on the model boundary we have an incoming velocity we prevent actual material ($c > 0$) from entering the model.

For modeling the friction between the work-piece and the rolls we implemented two friction laws: Coulomb's friction law and the constant friction law[1].

In what follows we discuss two technological applications of the modeling methodology we reviewed above.

In the first technological example we show that our simulation procedure is able to accurately describe the position of the work-piece / rolls *non-slip points* and to accurately predict the contact pressure distribution between the work-piece and the rolls: in Fig. 2 we present the results of 2D analyses of a roughing stand (Figs. 2.b and 2.c) and of a finishing stand (Figs. 2.d. and 2.e); both analyses were performed for different values of the friction coefficient "m" using the constant friction law. As expected [18] the roughing stand presents a *two-peaks type* contact pressure distribution while the finishing stand presents a *friction hill type* contact pressure distribution. Another important technological observation is that while in the roughing stand the position of the non-slip point is very sensitive to variations in the friction coefficient, in the finishing stand this sensitivity is quite low [12].

In the second technological example, summarized in Fig. 3, we present a 3D analysis: the modeling of the hot mandrel rolling of seamless steel tubes [9], where the effect of different roll grooves was investigated. The hypotheses were: rigid rolls, constant yield stress in the rolled tube, no inter-stand traction and Coulomb's friction. From the results we can make the following observations:

- The *groove shape # 1*, the narrower one, produces the highest value of axial stresses and therefore may produce a tensile failure in the rolled tube walls.
- The *groove shape # 2* offers the better solution among the three analyzed ones.
- The *groove shape # 3*, the one with the highest clearance between the work-piece and the rolls, produces an internal defect with the shape of ribs, which is unacceptable in the final product.

3 The Lagrangian rolls deformation model: its coupling to the Eulerian formulation

When modeling the rolling of steel coils, the assumption of elastic deformations for the work and back-up rolls (see Fig. 1) is normally used; hence we have to couple the Eulerian rigid-viscoplastic model of the plates deformation to a Lagrangian elastic model of the rolls deformation. The model of the rolls deformation includes the effect of the mechanical loads indicated in Fig. 1 and of the rolls heating due to their contact with the hot coils.

In Fig. 4 we present a scheme of the simulation procedure:

- The module **ROLLTEM** simulates the temperature evolution of the rolls due to their contact with the hot steel coils. In section 3.1 we discuss this simulation module.

- The module **ROLLEXP** simulates the thermal expansion of the rolls due to their temperature evolution. In section 3.2 we discuss this simulation module.
- The Eulerian model of the plate deformations described in the above section is coupled to **ROLLDEF**, a Lagrangian model of the rolls deformation described in section 3.3; the staggered iterative procedure developed for coupling the Eulerian and Lagrangian models is discussed in section 3.4.

3.1 Rolls temperature evolution

It is a 2D axisymmetric finite element model which simulates the work rolls thermal evolution. Even tough with an axisymmetric model we cannot determine the temperature peaks on the roll surface during rolling, these temperature peaks are only relevant for analyzing the thermal fatigue of the rolls surface and can be neglected when the objective is to evaluate the rolls thermal expansion.

On the finite element model that we show in Fig. 5 we impose the following boundary conditions:

- Contact between the work rolls and the steel coils: we use a convective heat transfer boundary condition with a coefficient h_p and a plate temperature T_p.
- Contact between the work rolls and the cooling water: we use a convective heat transfer boundary condition with a coefficient h_w and a plate temperature T_w.

For the hot rolling line shown in Fig. 6 the coefficients h_p and h_w were determined, for the F10 stand, by doing surface temperature measurements after the rolling schedule indicated in Fig. 7. The simulated temperature evolution of the work rolls is displayed in Fig. 8; in Fig. 8.h we show the agreement between the surface temperature measurements and the model predictions.

3.2 Rolls thermal expansion

The temperature distributions obtained using **ROLLTEM** are used in the module **ROLLEXP** to calculate the thermal expansion of the work rolls with an axisymmetric thermo-elastic model. As it is well known, it is important to use special care when solving thermo-elastic models to avoid the appearance of "parasitic stresses"; in our case we use the quadrilateral element QMITC [14].

With the temperature distributions determined using **ROLLTEM**, we calculate using **ROLLEXP** the work rolls thermal crown (see the Appendix for a definition of this terminology) at the instant at which the last coil of the considered rolling schedule exits the F10 stand (see Fig. 9)

3.3 Rolls deformation model

To simulate the rolls mechanical deformation we developed an elastic Lagrangian model which includes several geometrical nonlinearities: the contact between the work and back-up rolls and the flattening of the contact areas (Hertz problem). The numerical description of this phenomena is achieved using an *enhanced beam model* rather than a more expensive 3D geometrically nonlinear model of the rolls.

We discretize the work roll and its corresponding back-up roll using Hermitian beam elements that include the shear deformation [2]. In the area where contact between both rolls is possible we define pairs of matching nodes (one on each beam model) and between them we implement a *"node-to-node"* contact algorithm (usually the rolls are parallel and the matching nodes are located at the intersections between the beam axes and a common normal). We interpose between the matching nodes an ad hoc element that models the flattening of both surfaces in contact: our new *Hertz element.*

For the Hertz element connecting the nodes N_1 (corresponding to the work roll) and N_2 (corresponding to the back-up roll), using the nomenclature in Fig. 10, we define the initial gap,

$$\delta^0 = |\underline{r}_{N1} - \underline{r}_{N2}| - (R_1 + R_2) \tag{3}$$

please notice that due to the rolls mechanical and thermal crown the radius (R_1 and R_2) are variable node to node.

Loading the beam system (work + back-up rolls) with the forces determined in the Eulerian model plus the bending forces, we get the nodal displacements $\underline{U}_{N1}$ and $\underline{U}_{N2}$; hence , the gap undergoes a change, from δ^0 to $(\delta^0 + \Delta_U \delta)$,

$$\Delta_U \delta = (\underline{U}_{N2} - \underline{U}_{N1}) \cdot \underline{r} = u_2 - u_1 \tag{4}$$

where,

$$\underline{r} = \frac{\underline{r}_{N2} - \underline{r}_{N1}}{|\underline{r}_{N2} - \underline{r}_{N1}|} \quad .$$

The gap is also affected by the Poisson effect in the bent beams [16],

$$\Delta_\nu \delta = (\underline{U}^\nu_{N2} - \underline{U}^\nu_{N1}) \cdot \underline{r} = u^\nu_2 - u^\nu_1 \tag{5a}$$

$$|\underline{U}^\nu_{Ni}| = \frac{\nu \ |\underline{M}_i| \ R_i^2}{2 \ E_i \ I_i} \tag{5b}$$

the second equation was obtained from Ref. [24] and both displacements act in the direction of the bending displacements but with opposite sense. In the above equation,

ν : Poisson ratio,

$\underline{M}_i$**:** bending moment for the i-th roll at the considered section,

E_i, I_i **:** Young's modulus and inertia moment for the i-th roll at the considered section.

Therefore, the rolls bending makes the initial gap evolve from δ^0 to the value,

$$\delta^1 = \delta^0 + \Delta_U \delta + \Delta_\nu \delta \tag{6}$$

When $\delta^1 > 0$ the nodes N_1 and N_2 are not in contact and when $\delta^1 < 0$ both nodes are in contact and there is a contact force (P) among them. Being D the axial distance between equally spaced nodes on the roll axes and b the width of the contact zone for the N_1-N_2 Hertz element, we can write [21]

$$b = \sqrt{\frac{2\ P\ K_R\ C_E}{\pi\ D}} \tag{7a}$$

$$K_R = \frac{2\ R_1\ R_2}{R_1\ + R_2} \tag{7b}$$

$$C_E = \frac{1-\nu_1}{E_1} + \frac{1-\nu_2}{E_2} \tag{7c}$$

We approximate the radial displacement of each roll surface *(flattening)* with the superposition of the elastic solutions of a semi-infinite solid ($z \geqslant 0$) loaded with distributed loads, p_n, acting on the area $S_n = D\, b_n$ in the plane $z = 0$. Using a Cartesian coordinate system centered in the area S_n the solution of the Boussinesq problem is [19],

$$w(x,y,z) = \sum_{n=1}^{N_{Hertz}} \frac{(1+\nu)}{2\ E\ \pi} \int\int_{S_n} p_n(x\prime, y\prime) \left[\frac{2\ (1-\nu)}{r} + \frac{z^2}{r^3}\right]\ dx\prime dy\prime \tag{8a}$$

$$r = \sqrt{(x-x\prime)^2 + (y-y\prime)^2 + z^2} \tag{8b}$$

$$p_n = \frac{P_n}{S_n} \tag{8c}$$

where N_{Hertz} is the number of Hertz elements.

With a close form solution we can calculate the integral on the r.h.s. of Eqn. (8a). The contribution of the j-roll ($j = 1,2$) to the flattening of the i-th Hertz element is,

$$a_{ij} = w_{ij}(0,0,0) - w_{ij}(0,0,R)\ \ . \tag{9}$$

The flattening of the i-th Hertz element is the addition of all the contributions we evaluated above; hence, the gap evolves from its original value to a final value,

$$\delta_i = \delta_i^0 + \Delta_U \delta_i + \Delta_\nu \delta_i + a_{i1} + a_{i2} = \delta_i^1 + a_{i1} + a_{i2} \tag{10a}$$

the contact condition imposes,

$$\delta_i = 0 \tag{10b}$$

In order to determine the total rolls deformation it is necessary to add, to the deformation calculated as detailed above, the flattening of the work rolls due to their contact with the rolled steel plates. For this purpose we use Eqn. (8a) integrated over the contact surface between the work roll and the rolled steel plate.

3.4 Coupling of the Eulerian and Lagrangian models

The coupling between the Eulerian rigid-viscoplastic model that describes the rolled coils deformation and the Lagrangian elastic model that describes the rolls deformation is performed via an iterative staggered scheme that we describe in what follows:

1. $k = 0$
2. *Assume for the rolls a trial deformation* $\longrightarrow (\underline{U}^{(k)}_{rolls})$
3. $k = k + 1$
4. *Keep fixed* $(\underline{U}^{(k-1)}_{rolls})$ *and solve the equilibrium equations in Box I.*
5. *From the above step get the loads imposed by the rolled coils on the rolls* $(\underline{R}^{(k)})$
6. *Solve the enhanced beam model under the loads* $\underline{R}^{(k)}$ *and the bending forces* $\longrightarrow \underline{U}^{(k)}_{rolls}$
7. $IF \ \frac{\left\| \underline{U}^{(k)}_{rolls} - \underline{U}^{(k-1)}_{rolls} \right\|_2}{\left\| \underline{U}^{(k)}_{rolls} \right\|_2} \leq UTOL$

   $THEN \longrightarrow CONVERGENCE$

   $ELSE \longrightarrow GO\ TO\ 3$

Box II: Coupling between the Eulerian and Lagrangian models

3.5 Qualification of the coupled model for a finishing stand

In a previous publication [9] we tested our model comparing its predictions with industrial measurements on the roughing stand R4 (Fig. 6). Even tough the test was successful, it was not decisive because in a roughing stand the numerical values of the flattening (a_{ij} in Eqn. (9)) are negligible as compared with the plate thickness reductions; hence, the model of a roughing stand is not suitable for a complete validation of our rolling simulation procedure.

In order to validate the complete rolling simulation procedure schematized in Fig. 4, we now compare its predictions for the case of the F10 stand in Fig.

6, with industrial on-line measurements carried on at the hot rolling steel mill of SIDERAR (San Nicolás, Argentina).

To perform the simulation we considered the following data:

- Plate profile at the stand entrance with a *relative crown* (*crown / thickness*) equal to the relative crown measured at the stand exit[2].
- Constant plate yield stress (20.5 kg/mm^2) calculated matching the measured separating force (829 *ton*) with the numerically predicted separating force.
- Bending forces ($F_1 = 16ton$; $F_2 = 0$) (see Fig. 1)

In Fig. 11 we compare the numerically predicted thickness distribution across the plate width with on-line measurements of the thickness distributions across the plate width. As it can be observed the numerical results are quite satisfactory; hence, we can consider that the ability of our finite element procedure for modeling a rolling stand has been established.

It is important to remark that if instead of calculating the work-piece yield stress as described above, we calculate it using the expressions in Ref. [20] with a stand temperature of $920°C$, our model produces a total separating force 25% higher that the measured one; even tough this discrepancy is in the order of the errors reported in Refs.[5]to[7]for models that do not include the thermo-mechanical-metallurgical coupling, it is too high for our purpose: the prediction of the plate profile. Of course, when implementing a coupled thermo-mechanical-metallurgical model we will not need to calibrate the yield stress using the measured separating forces.

4 Technological applications

In this section we describe an application of our simulation procedure developed to explore the technological windows of the steel coils hot rolling mill of SIDERAR (San Nicolás, Argentina). For the stand F10 we analyze the following case:

- Roll profiles as per Fig. 12. We considered two cases for the work rolls: a total positive *roll crown* [3] of 50μ and a total negative roll crown of 150μ.
- Incoming plate crown: 48μ.
- Plate thickness at the stand entrance: 2.43 mm.
- Plate thickness at the stand exit: 2.00 mm.
- Plate width: $1,250$ mm.

[2] See Appendix

[3] See the Appendix for a definition of "roll crown"

- Back-up rolls Young's modulus: $21,000\ kg/mm^2$.
- Work rolls Young's modulus: $18,000\ kg/mm^2$.
- Poisson coefficient: 0.3 (both rolls)

In Fig. 13 we present the obtained results. It is interesting to notice that:

Work rolls crown [μ]	**Plate crown** [μ]
50	48.22
-150	203.25

Table I: Effect of the work rolls crown on the plate profile

In the first case the change in the plate *relative crown*[4] is small and the corresponding separating forces are almost constant across the plate width; on the other hand, in the second case the change in the plate relative crown is quite important and the corresponding separating forces are not uniform across the plate width.

5 Conclusions

For an accurate simulation of steel rolling processes a number of models, that describe different physical phenomena need to be coupled. In this paper we present a model that couples the deformation of the work-piece (steel tubes or steel coils) to the thermal evolution of the rolls, its thermal expansion and its mechanical deformation.

The model has been validated comparing its predictions to a set of industrial measurements and has also been used for the analysis of actual technological problems.

Our next step will be to also couple to the simulation procedure a model of the work-piece thermo-metallurgical evolution[4].

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[4] See Appendix

References

[1] W.Backofen, *Deformation Processing*, Addison-Wesley, 1972.

[2] K.J.Bathe, *Finite Element Procedures*, Prentice Hall, 1996.

[3] M.A.Cavaliere, M.B.Goldschmit and E.N.Dvorkin, "3D modeling of bulk metal forming processes via the flow formulation and the pseudo - concentrations technique", *Proceedings Fifth Int. Conf. on Computational Plasticity*, (Ed. D.R.J.Owen et al), CIMNE, 1997

[4] M.A.Cavaliere, M.B.Goldschmit and E.N.Dvorkin,"On the solution of thermo-mechanical problems vis the pseudo-concentrations technique", *Proceedings ECCOMAS 2000,* (Ed. E.Oñate et al), CIMNE, 2000.

[5] C.Devadas, I.V.Samarasekera and E.B.Hawbolt, "The thermal and metallurgical state of steel strip during hot rolling: Part I. Characterization of heat transfer", *Metallurgical Transactions*, **22A**, pp.307-319, 1991.

[6] C.Devadas, D.Baragar, G.Ruddle, I.V.Samarasekera and E.B.Hawbolt, "The thermal and metallurgical state of steel strip during hot rolling: Part II. Factors influencing rolling loads", *Metallurgical Transactions*, **22A**, pp.321-333, 1991.

[7] C.Devadas, I.V.Samarasekera and E.B.Hawbolt, "The thermal and metallurgical state of steel strip during hot rolling: Part III. Microstructural evolution", *Metallurgical Transactions*, **22A**, pp.335-349, 1991.

[8] E.N.Dvorkin, M.A.Cavaliere and M.B.Goldschmit, "A three field element via augmented Lagrangian for modelling bulk metal forming processes", *Comput. Mechs.*, **17**, pp.2-9, 1995.

[9] E.N.Dvorkin, M.A.Cavaliere, M.B.Goldschmit and P.M.Amenta, "On the modeling of steel product rolling processes", *Int.J.Forming Processes (ESAFORM)*, **1**, pp.211-242, 1998.

[10] E.N.Dvorkin, M.A.Cavaliere, M.G.Zielonka and M.B.Goldschmit, "New developments for the modeling of metal rolling processes", *Proc. European Conference on Computational Mechanics (ECCM'99),* München, Germany, 1999.

[11] E.N.Dvorkin, M.B.Goldschmit, M.A.Cavaliere and P.M.Amenta, "On the modelling of bulk metal forming processes", *Proc. Second ECCOMAS (European Community on Computational Methods in Applied Sciences) Conference on Num. Methods in Engrg.*, John Wiley & Sons, 1996.

[12] E.N.Dvorkin, M.B.Goldschmit, M.A.Cavaliere, P.M.Amenta, O.Marini and W.Stroppiana, "2D finite element parametric studies of the flat rolling process", *J.Materials Processing Technology*, **68**, pp.99-107, 1997.

[13] E.N.Dvorkin and E.G.Petöcz, "An effective technique for modelling 2D metal forming processes using an Eulerian formulation", *Engrg. Comput.*, **10**, pp.323-336, 1993.

[14] E.N.Dvorkin and S.I.Vassolo, "A quadrilateral 2D finite element based on mixed interpolation of tensorial components", *Engrg. Comput.*, **6**, pp.217-224, 1989.

[15] V.Ginzburg, *High-Quality Steel Rolling: Theory and Practice*, Marcel Decker Inc., 1993.

[16] A.Hacquin, P.Montmitonnet and J.P.Guillerault, "Experimental validation of a rolling stand elastic deformation model", *J.Materials Processing Technology*, **45**, pp.199-206, 1994.

[17] T.J.R.Hughes, *The Finite Element Method*, Prentice-Hall, 1987.

[18] G.Li and S.Kobayashi, "Rigid-plastic finite element analysis of plane strain rolling", *J.Engng. Industry*, **104**, pp.55-63, 1982.

[19] A.E.H.Love, *A Treatise on the Mathematical Theory of Elasticity*, Dover Publications, 1944.

[20] M.Pietrzyk and J.Lenard, *Thermal-Mechanical Modelling of the Flat Rolling Process*, Springer-Verlag, 1991.

[21] R.J.Roark and W.C.Young, *Formulas for Stress and Strain* (5th. Edition), McGraw-Hill, 1975.

[22] E.Thompson, "Use of the pseudo-concentrations to follow creeping viscous flows during transient analysis", *Int.J.Num.Meth.Fluids*, **6**, pp.749-761, 1986.

[23] E.Thompson and R.E.Smelser, "Transient analysis of forging operations by the pseudo-concentrations method", *Int.J.Num.Meth.Engrg.,* **25**, pp.177-189, 1988.

[24] C.T. Wang, *Applied Elasticity*, McGraw-Hill,1953.

[25] O.C.Zienkiewicz, P.C.Jain and E.Oñate, "Flow of solids during forming and extrusion: some aspects of numerical solutions", *Int.J.Solid Struct.*, **14**, pp.15-28, 1977.

[26] O.C.Zienkiewicz and R.L.Taylor, *The Finite Element Method* (4th. Edition), McGraw-Hill, 1989.

A Coils rolling terminology

In this Appendix we are going to briefly comment some concepts pertaining to steel coils hot rolling technology, that have been used in the main body of this paper.

A.1 Plate profile and plate crown

The transversal section of a rolled steel coil is usually not a rectangle but it has a shape similar to the one schematized in Fig. A.1, this shape is referred to as the *plate profile*. In order to have a quantitative measure of the difference between the plate thickness at the center of its transversal section and near its edges the *plate crown* is defined; in the same figure we indicate this definition.

A.2 Plate flatness

Since the transversal section of a plate is of variable thickness, it is apparent that during rolling different fibers located at different locations across the plate will undergo different elongations; hence, due to the plate continuity, some fibers will be in a tensile state and others in a compressive state. It is well known that the compressed parts may buckle and therefore the plate may loose its *flatness*.

To quantify the tendency towards buckling at a given stand the following technological parameter is used [15]:

$$\delta = \frac{plate\ crown\ upstream\ the\ stand}{plate\ thickness\ upstream\ the\ stand} - \frac{plate\ crown\ downstream\ the\ stand}{plate\ thickness\ downstream\ the\ stand}$$

The parameter δ is the difference between the *plate relative crowns* upstream and downstream the stand. If we neglect the lateral spreading of the plate (a valid assumption for the last finishing stands) then,

- $\delta < 0$ indicates a tendency towards buckling at the plate edges,
- $\delta > 0$ indicates a tendency towards buckling at the plate center.

Therefore, the objective of a stand set-up is a rolling condition with $\delta = 0$ in order to produce a flat plate. However, it has been experimentally defined a range inside which it can be assured the flatness of the plate [15]:

$$-80 \left[\frac{downstream\ thickness}{plate\ width}\right]^{1.86} < \delta < 40 \left[\frac{downstream\ thickness}{plate\ width}\right]^{1.86}$$

A.3 Rolls profile and crown

The work rolls and sometimes also the back-up rolls are not straight cylinders, usually the cylinder generatrices have a shape similar to the ones indicated in

Fig. A.2 (*roll profiles*), to compensate the bending of the rolls and therefore produce a plate with a smaller crown. The number used to define a roll profile is the *roll crown* whose definition is also indicated in the same figure.

$$total\ rolls\ crown = rolls\ mechanical\ crown + rolls\ thermal\ crown$$